MPM 1D9

Grade 9 Pre-AP Math
Unit 5 – Measurement & Geometry Justifications – Practice Test

Victim: Mr. Solutions

KU	APP	TIPS	COM
12 /12	16 /16	14 /14	14/14

Modified True or False (6 KU)

Indicate whether each statement is ***true*** or ***false***. If false, ***change*** the underlined part to make the statement true.

1. F ✓ A cylinder of fixed area has maximal volume if its radius equals its height. ✓ = ½ mark **Change:** diameter ✓
2. F ✓ If the radius of a circle is doubled, its area is doubled. **Change:** quadrupled ✓
3. T ✓ A rectangle of fixed area has minimal perimeter if all sides have equal length. **Change:** ✓ (left blank)

Multiple Choice (5 KU)

For questions 4 to 8, select the ***best*** answer. Write the letter of your choice in the provided blank space.

4. b ✓ One container is in the shape of a square-based prism and another is in the shape of a cylinder. If the containers have exactly the same base area and exactly the same height then which statement is true?

   (a) ✗ The cylindrical container has a greater volume. (b) ✓ The containers have the same volume.
   (c) ✗ The cylindrical container has a smaller volume. (d) None of the above.

   Volume of prism or cylinder = A_{Base} (height)

5. d ✓ One container is in the shape of a cylinder and another is in the shape of a cone. If the containers have exactly the same volume then which statement is true?

   ✓ (a) The containers have the same base area but the cylinder's height is one-third the cone's height.
   ✗ (b) The containers have the same base area but the cylinder's height is three times the cone's height.
   ✓ (c) The containers have the same height but the cone's base area is three times the cylinder's base area.
   (d) ✓ Both (a) and (c).

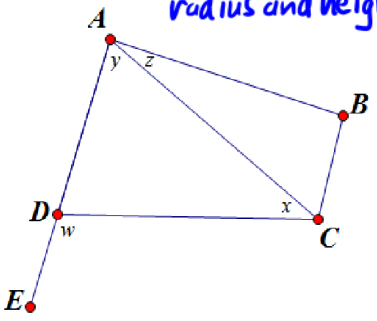
   $V_{cone} = \frac{1}{3} A_{base}$ (height) = ⅓ (volume of cylinder with same radius and height)

6. b ✓ Refer to the geometric figure at the right. Which statement is true?

   (a) ✗ $w = y + z$ (b) $w = x + y$ (c) ✗ $z = x$ (d) ✗ Both (b) and (c)

   only true if AB ∥ DC

   ← because of EAT



7. d ✓ In the geometric figure at the right, what is true about quadrilateral *ABCD*?

   (a) ✗ It's a trapezoid. (b) ✗ It's a parallelogram.
   (c) ✗ It's a doggie face. (d) ✓ None of the above.

8. c ✓ Which statement is ***not*** true?

   ✓ (a) President Garfield created a clever proof of the Pythagorean Theorem.
   ✓ (b) The area of a square built upon the hypotenuse of a right triangle is equal to the sum of the areas of the squares built upon the other two sides.
   ✗ (c) The Pythagorean Theorem applies to all triangles.
   ✓ (d) In a right triangle, the hypotenuse is always opposite the 90° angle.

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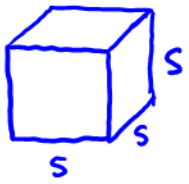
Full Solutions/Explanations

9. Ms. Tatelman would like to build a sound-proof detention centre for those who have difficulties appreciating the "art of silence." If the detention centre must take the shape of a square-based prism and its volume must be 64 m^3, find the ***dimensions*** of the "prism of silence" that could be built with the least amount of material. (For this question it is sufficient simply to state what shape would minimize the amount of material needed. It is not necessary to develop equations or sketch graphs.) **(4 KU)**

Least material
→ minimal surface area ✓
→ square-based prism must be a cube ✓



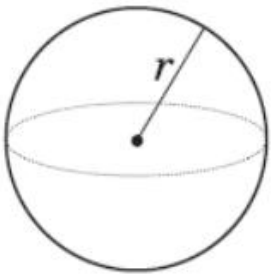
Let s represent the side length of the cube. Since the volume must be 64 m^3,

$s^3 = 64$ ✓

$\therefore s = \sqrt[3]{64} = 4$ ✓

To construct the "prism of silence" with the least material possible, the dimensions must be 4m x 4m x 4m.

10. Krisnavee uses her mighty lungs (developed through a lifetime of yakking) to help Abi inflate a spherical balloon. Once Abi ran out of breath, Krissnavee took over and blew so hard that the balloon's radius tripled. What happened to the surface area of the balloon when its radius tripled? **(4 APP)**



$A = 4\pi r^2$

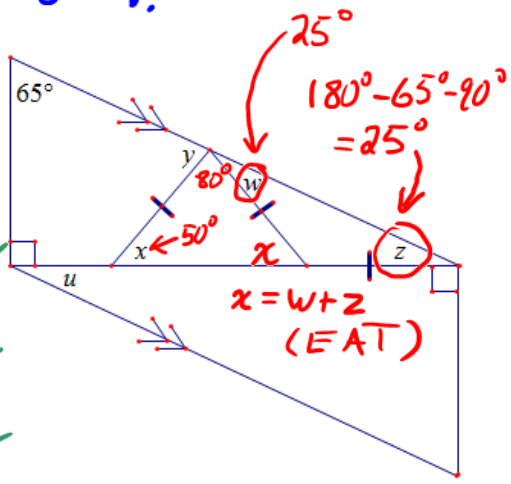
Abi → blew up ballon to a radius of r units
$\therefore A_{Abi} = 4\pi r^2$ ✓

Krissnavee → blew up the balloon to a radius of 3r
$\therefore A_{Kriss.} = 4\pi(3r)^2$ ✓ $= 4\pi(9r^2) = 9(4\pi r^2) = 9A_{Abi}$ ✓

Krissnavee's mighty lungs made the balloon's surface area 9 times larger !! ✓

11. Complete the following table. **(5 APP)**

Angle Measure	*Justification (i.e. State Reasons)*
$u =$ 25°	u=z (z, u) ✓
$w =$ 25°	w=z (base angles of an isosceles triangle) ✓
$x =$ 50°	Base angles of an isosceles triangle are equal and x=w+z (EAT) ✓
$y =$ 75°	y+80°+25°=180° (straight angle) ✓
$z =$ 25°	180°-90°-65°=25° ✓



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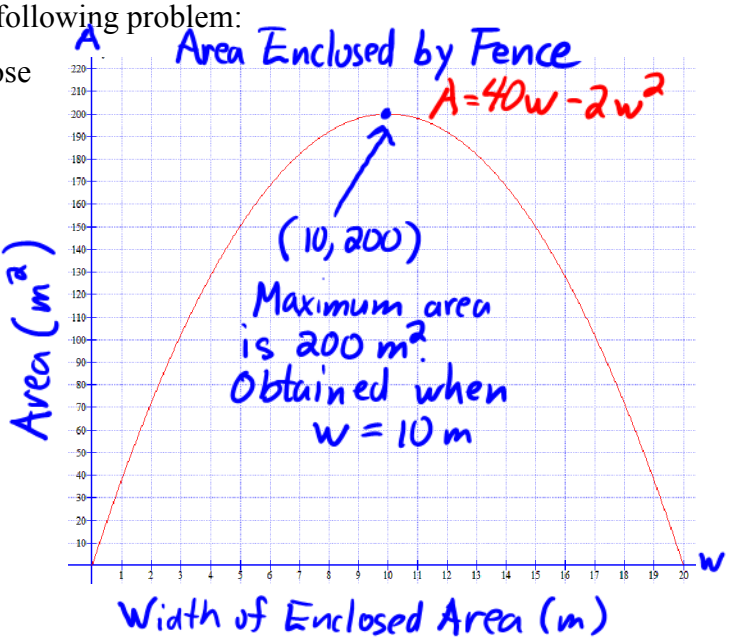
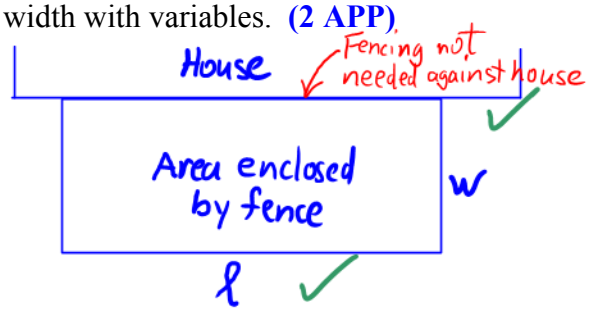
12. The graph shown at the right can be used to solve the following problem:

You have 40 m of fencing and you would like to enclose a rectangular region of ***greatest possible area*** against one side of your house. What dimensions should the rectangle have?



(a) Draw a diagram of the rectangular region showing how it is fenced off. Label the length and the width with variables. **(2 APP)**



(b) Write an equation that relates the length and the width of the rectangular region. **(2 APP)**

$2w+\ell = 40$

(c) Write an equation that relates the area of the rectangular region to its width. **(3 APP)**

$A = \ell w$

$= (40-2w)w$

$\therefore \quad A = w(40-2w) = 40w - 2w^2$

Note:
Since $2w+\ell = 40$,
$\ell = 40-2w$

(d) Label the graph given above. Include a title and labels for both axes. In addition, label the curve with its equation. **(4 TIPS)** See graph

(e) Now state conclusions. Specifically, state the length and width that maximize the area and explain how you obtained these values. **(4 COM)**

As shown in the graph, the point (10, 200) is the point on the graph with the largest possible area.
Therefore, the maximal area is 200 m².
It is obtained when $w = 10$ m
and $\ell = 40-2w = 40-2(10) = 20$ m.

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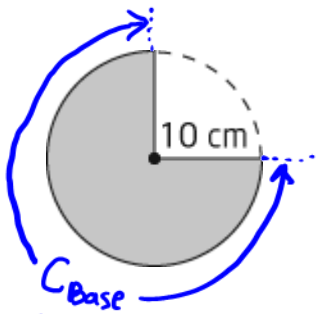
13. A cone is formed from a circle of radius 10 cm with a sector of 90° cut out. **(6 TIPS)**



(a) What is the slant height of the cone?

The slant height of the cone is equal to the radius of the sector, which is 10 cm.

(b) What is the radius of the cone?

Let C_{Base} represent the circumference of the cone's base and let r represent the radius of the cone.

$\therefore C_{Base} = \text{arc length of sector} = \frac{3}{4}[2\pi(10)] = \frac{60\pi}{4}$

$\therefore C_{Base} = 15\pi$

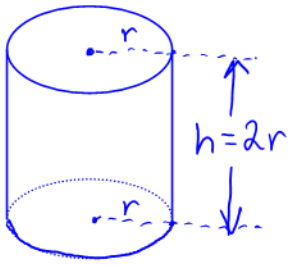
$\therefore 2\pi r = 15\pi$

$\therefore r = \frac{15\pi}{2\pi} = \frac{15}{2}$

The radius of the cone must be $\frac{15}{2} = 7.5$ cm.

14. Big Bran breakfast cereal is sold in a cylindrical box has whose height is double the radius. The manufacturer also sells the cereal in a cylindrical box that has dimensions three times those of the smaller box. Compare the volume of the two boxes and explain your answer. **(4 TIPS)**

Small Box



Large Box

radius = $3r$

height = $3(2r)$
$= 6r$

Therefore,

$$\frac{V_{\text{large box}}}{V_{\text{small box}}} = \frac{\pi(3r)^2(6r)}{\pi r^2(2r)}$$

$$= \frac{\pi(9r^2)(6r)}{2\pi r^3}$$

$$= \frac{54\pi r^3}{2\pi r^3}$$

$$= 27$$

Rough Work:

$\frac{54}{2}\left(\frac{\pi r^3}{\pi r^3}\right) = 27(1)$
$= 27$

The volume of the large box is 27 times greater than the volume of the small box.

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