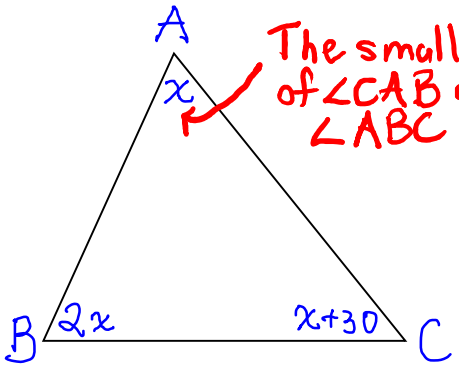
Solutions: TIPS Problems Involving Equations

1. In ΔABC, the measure of $\angle ABC$ is double the measure of $\angle CAB$. The measure of $\angle BCA$ is 30° greater than the measure of the smaller of the other two angles. Find the measure of each angle.



The smaller of $\angle CAB$ and $\angle ABC$

Let x represent the measure of $\angle CAB$.
Then, $2x$ represents the measure of $\angle ABC$
and $x+30$ represents the measure of $\angle BCA$.

(The sum of the interior angles of a triangle) is 180°

$$x + 2x + x + 30 = 180$$
$$\therefore 4x + 30 = 180$$
$$\therefore 4x + 30 - 30 = 180 - 30$$
$$\therefore 4x = 150$$
$$\therefore \frac{4x}{4} = \frac{150}{4}$$
$$\therefore x = 37.5$$

$\angle CAB = 37.5^\circ$
$\angle ABC = 2(37.5^\circ) = 75^\circ$
$\angle BCA = 37.5^\circ + 30$
$= 67.5^\circ$

2. Naquan is saving nickels and dimes in a jar. The jar contains 10 more nickels than dimes. Altogether, the value of the coins is $16.25. How many nickels and dimes are in the jar?

Coin	*Value of One Coin*	*Number of Coins*	*Value of Coins*
Dime	$0.10	d	$0.10d$
Nickel	$0.05	$d+10$	$0.05(d+10)$
Total	N/A	$d+d+10$	$16.25

Not relevant to solving this problem

(Value of dimes) + (Value of nickels) is 16.25

$$0.10d + 0.05(d+10) = 16.25$$
$$\therefore 0.10d + 0.05d + 0.5 = 16.25$$
$$\therefore 0.15d + 0.5 - 0.5 = 16.25 - 0.5$$
$$\therefore \frac{0.15d}{0.15} = \frac{15.75}{0.15}$$
$$\therefore d = 105$$
$$\therefore d + 10 = 115$$

Nickel = $5^¢$ = $0.05

Dime = $10^¢$ = $0.10

Conclusion
There are 105 dimes and 115 nickels in the jar.

Check
$105(0.10) + 115(0.05) = 10.5 + 5.75$
$= 16.25$ ✓

3. Solution A is 50% hydrochloric acid by volume, while solution B is 75% hydrochloric acid by volume. How many litres of each solution should be used to make 100 litres of a solution which is 60% hydrochloric acid by volume?

"a" litres of solution A
100 litres altogether
∴ 100 − a litres of solution B

Solution (of Problem)

Solution	*% of acid*	*Volume of the Solution (L)*	*Volume of Acid in the Solution (L)*
Solution A	50% = 0.5	a	$0.5a$
Solution B	75% = 0.75	$100-a$	$0.75(100-a)$
Mixture	60% = 0.6	100	$0.6(100)=60$

HCl = Hydrochloric Acid

(Volume of HCl in Solution A) + (Volume of HCl in Solution B) = (Total Volume of HCl in the mixture)

$$0.5a + 0.75(100-a) = 60$$

$$\therefore 0.5a + 75 - 0.75a = 60$$

$$\therefore -0.25a + 75 - 75 = 60 - 75$$

$$\therefore \frac{-0.25a}{-0.25} = \frac{-15}{-0.25}$$

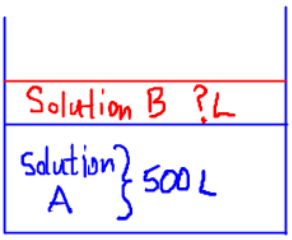
$$\therefore a = 60$$

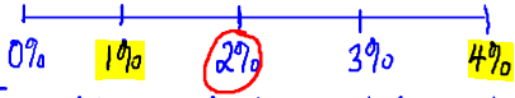
$$\therefore 100 - a = 100 - 60 = 40$$

The mixture should contain 60 L of solution A and 40 L of solution B.

4. To make lower-fat chocolate frozen yogurt, chocolate milk containing 2% butterfat is needed. To obtain the required percentage of butterfat, chocolate milk containing 4% butterfat is mixed with 500 litres of chocolate milk containing 1% butterfat.

(a) Without performing any calculations, predict whether the mixture will contain more of the 1% chocolate milk or more of the 4% chocolate milk. Explain.





I would expect the mixture to contain more of the 1% milk because 1% is closer to 2% than 4% is

(b) How many litres of the 4% chocolate milk are needed to create the required mixture? What is the total volume of the mixture?

Solution

Let x represent the number of litres of the 4% chocolate milk that must be mixed with 500 L of the 1% chocolate milk to produce a mixture containing 2% butterfat.

Type of Chocolate Milk	*Volume (L)*	*Amount of Butterfat in Given Volume (L)*
1% butterfat	500	$0.01(500) = 5$
4% butterfat	x	$0.04x$
2% butterfat mixture	$x+500$	$0.02(x+500)$

(Volume of butterfat in mixture from 1% milk) + (Volume of butterfat in mixture from 4% milk) = (Total volume of butterfat in the mixture)

$$5 + 0.04x = 0.02(x+500)$$

$$\therefore 5 + 0.04x = 0.02x + 10$$

$$\therefore 5 + 0.04x - 0.02x - 5 = 0.02x + 10 - 0.02x - 5$$

$$\therefore 0.02x = 5$$

$$\therefore \frac{0.02x}{0.02} = \frac{5}{0.02}$$

$$\therefore x = 250$$

To create the required mixture, 250 L of 4% chocolate milk must be added to 500 L of 1% chocolate milk, for a total volume of 750 L.

(c) Does your answer in part (b) agree with the prediction that you made in part (a)? What can you conclude from this?

My answer agrees with the prediction from part (a), which means that it is likely to be correct.