SOLVING LINEAR SYSTEMS BY ELIMINATION

Overview
- Multiply or divide both sides by the same value to produce like terms with the same or opposite coefficient
- Add or subtract equations to eliminate one of the variables

Examples

Use the method of elimination to solve each of the following systems:

A. $\begin{cases} x + 2y = 20 \quad ① \\ 2x + 3y = 12 \quad ② \end{cases}$

> At this stage, it's not possible to eliminate any variables because the equations don't have any common terms.

$① × 2, \quad 2x + 4y = 40 \quad ③$

> Multiply both sides of equation ① by 2. Now equations ② and ③ have a common term of $2x$.

$\begin{aligned} ③ - ② \quad & 2x + 4y = 40 \\ & -(2x + 3y = 12) \\ \hline & y = 28 \end{aligned}$

> This really means $2x + 4y - (2x + 3y) = 40 - 12$

Sub. in ①,

$x + 2(28) = 20 \rightarrow x + 56 = 20 \rightarrow x = -36.$

The solution of this system is $(x, y) = (-36, 28)$.

B. $\begin{cases} 2x + 3y = 17 \quad ① \\ 9x - 4y = 24 \quad ② \end{cases}$

$① × 4, \quad 8x + 12y = 68 \quad ③$
$② × 3, \quad +(27x - 12y = 72) \quad ④$

$③ + ④, \quad 35x = 140$

$\therefore x = 4$

Sub. in ①, $2(4) + 3y = 17 \rightarrow y = 3.$
The solution is $(x, y) = (4, 3)$

Exercises

Use elimination to solve the following linear systems. Verify each solution graphically.

(a) $\begin{cases} x - y = 2 \\ 4x - 6y = 4 \end{cases}$

(b) $\begin{cases} -5x + 3y = 1 \\ -7x - 2y = -42 \end{cases}$

(c) $\begin{cases} 5x + 8y = 13 \\ 6x - 15y = -9 \end{cases}$

(d) $\begin{cases} 13x - 5y = 7 \\ -2x + 7y = -30 \end{cases}$