Unit 1 – Number Sense and Algebra

Examples of Number Sense

1. Why is ***division by zero undefined***?

$$100 \div 20$$

MEANS

"How many groups of 20 can be formed from 100 objects?
Since 5 groups of 20 can be formed, the answer is 5.

$$100 \div \frac{1}{2}$$

MEANS

"How many groups of $\frac{1}{2}$ can be formed from 100 objects?

Since 200 groups of $\frac{1}{2}$ can be formed, the answer is 200.

$$100 \div 0$$

MEANS

"How many groups of 0 can be formed from 100 objects? Any number of groups of zero will always add up to zero! Clearly then, it is ***impossible*** to make 100, or any other value for that matter, from groups of zero! Thus, we say that division by zero is ***undefined***.

LANGUAGE HAS MEANING!

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MATH IS A LANGUAGE!

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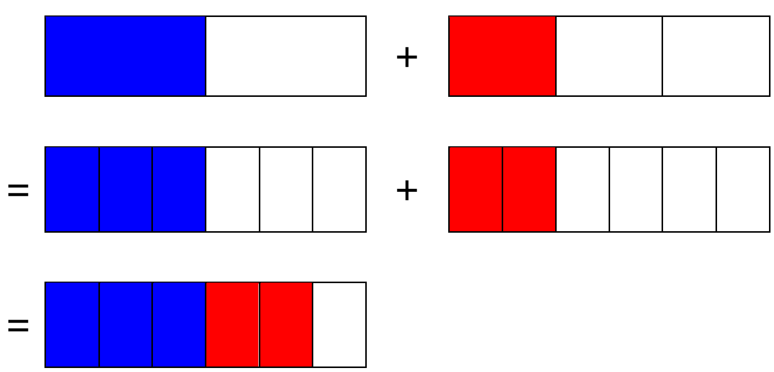
THEREFORE, MATH HAS MEANING!

2. Explain why $\frac{1}{2}+\frac{1}{3}$ ***cannot possibly be equal to*** $\frac{2}{5}$.



$\frac{1}{2}+\frac{1}{3}$

$=\frac{3}{6}+\frac{2}{6}$

$=\frac{5}{6}$

The diagrams at the right clearly show that $\frac{1}{2}+\frac{1}{3}$ must equal $\frac{5}{6}$! Even without the diagrams, we could ***reason*** that $\frac{1}{2}+\frac{1}{3}$ must be ***greater than*** $\frac{1}{2}$ but $\frac{2}{5}$ is obviously ***less than*** $\frac{1}{2}$!

Definition of "Undefined"

Whenever mathematicians ***cannot find a way to give a meaning*** to a mathematical term, operation or other mathematical concept, they say that it is ***undefined***.

Questions

1. What is an integer?

2. What is a rational number? Why are such numbers called rational?

+(+) add a positive: GAIN
-(-) subtract a negative: GAIN

+(-) add a negative: LOSS
-(+) subtract a positive: LOSS

Gains more than losses: + answer
Losses more than gains: - answer

Rule for Determining the Sign of the Answer when Multiplying and Dividing

Two Numbers
Signs Same: + answer
Signs Different: - answer

More than Two Numbers
Even # of Negatives: + answer
Odd # of Negatives: - answer

Multiplying Fractions

- Multiply the numerators and multiply the denominators.
- If possible, reduce to lowest terms.

OR

- Reduce first (vertically and diagonally)
- Multiply the numerators and multiply the denominators.

$$\frac{3}{10}\left(\frac{8}{15}\right)=\frac{24}{150}=\frac{4}{25} \quad \textbf{OR} \quad \frac{\cancel{3}^{1}}{\cancel{10}_{5}}\left(\frac{\cancel{8}^{4}}{\cancel{15}_{5}}\right)=\frac{4}{25}$$

Dividing Fractions

- ***Do not*** change the 1st fraction.
- Change ÷ to ×.
- Find the reciprocal of the 2nd fraction (i.e. "***flip***").
- **Summary:** Multiply by the reciprocal

$$\frac{3}{10}\div\frac{8}{15}=\frac{3}{10}\times\frac{15}{8}=\frac{45}{80}=\frac{9}{16}$$

Adding/Subtracting Fractions

- Express each fraction with a common denominator.
- Add/subtract the numerators.
- Keep the denominator!
- If possible, reduce to lowest terms.

$$\frac{3}{10}+\frac{8}{15}=\frac{9}{30}+\frac{16}{30}=\frac{25}{30}=\frac{5}{6}$$

3. ***Evaluate*** each of the following expressions ***without*** using a calculator.

(a) $-3+5$ **(b)** $-3-5$ **(c)** $-3+(-5)$ **(d)** $-3-(-5)$

(e) $-3+5-6+1$ **(f)** $-3-5-6+1$ **(g)** $-3-(-5)+(-6)+1$ **(h)** $3+(-5)+(-6)-(-1)$

4. ***Evaluate*** each of the following expressions ***without*** using a calculator.

(a) $-3(5)$ **(b)** $-3(+5)$ **(c)** $-3(-5)$ **(d)** $3(-5)$

(e) $-3(5)(-6)(1)$ **(f)** $-3(5)(-6)(-1)$ **(g)** $3(5)(6)(-1)$ **(h)** $-3(-5)(-6)(-1)$

5. ***Evaluate*** each of the following expressions ***without*** using a calculator.

(a) $\frac{36}{-12}$ **(b)** $\frac{-36}{-12}$ **(c)** $\frac{+49}{+7}$ **(d)** $\frac{-64}{16}$

6. Draw diagrams to represent each of the following fractions.

(a) $\frac{3}{5}$ **(b)** $\frac{5}{3}$ **(c)** $2\frac{9}{10}$ **(d)** $\frac{6}{2}$

7. ***Evaluate*** each of the following ***expressions***.

(a) $\frac{3}{5}-\frac{2}{5}$ **(b)** $\frac{-5}{3}+\frac{5}{6}$ **(c)** $-\frac{5}{14}+\left(-\frac{8}{21}\right)$ **(d)** $-\frac{5}{14}-\left(-\frac{8}{21}\right)$

8. ***Evaluate*** each of the following ***expressions***.

(a) $\frac{3}{5}\left(-\frac{2}{5}\right)$ **(b)** $\frac{-5}{3}\left(+\frac{5}{6}\right)$ **(c)** $-\frac{5}{14}\left(-\frac{8}{21}\right)$ **(d)** $\frac{5}{14}\left(-\frac{8}{21}\right)$

9. ***Evaluate*** each of the following ***expressions***.

(a) $\frac{3}{5}\div\left(-\frac{2}{5}\right)$ **(b)** $\frac{-5}{3}\div\left(+\frac{5}{6}\right)$ **(c)** $-\frac{5}{14}\div\left(-\frac{8}{21}\right)$ **(d)** $\frac{5}{14}\div\left(-\frac{8}{21}\right)$

1 2 3 4
B E DM AS
Division/Multiplication - tied, left-to-right
Addition/Subtraction - tied, left-to-right

Terms are separated by + and - signs.

Separate each expression into terms. Then apply the operations in the correct order.

10. ***Evaluate*** each of the following ***expressions*** by applying the operations in the correct order.

(a) $-20 \div \left(-4-(-8)\right)$

(b) $-20-4(-8)^2$

(c) $-20+\left(-4-(-2)^3\right)$

(d) $2(-7)-\dfrac{10}{2^2-3^2}+2(-3)^4$

(e) $-3\left[-2+2(6)-4(3)^3\right]^4$

(f) $\dfrac{-10+5(-3)}{\left[2-(-3)\right]^2}$

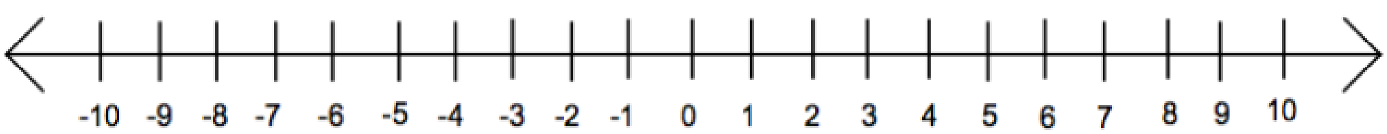
(g) $-\dfrac{5}{14}+\left(-\dfrac{8}{21}\right)\left(\dfrac{7}{-4}\right)$

(h) $-\dfrac{5}{3}\div\dfrac{10}{9}+\left(-\dfrac{8}{21}\right)\left(\dfrac{3}{-4}\right)^2$

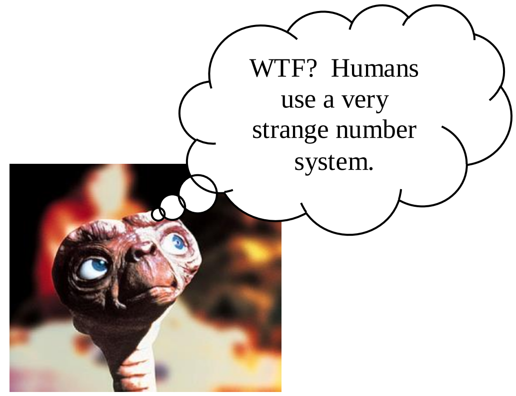
(i) $\dfrac{-10+5(-3)}{\left[2-(-3)\right]^2}-\left(\dfrac{-5}{3}\right)\left(+\dfrac{5}{6}\right)$

11. Use the given number line to arrange the following numbers in order from smallest to largest.

8.3, –2.9, 0.05, 6.4, –0.8, –2.2 –8.3, 0.5



12. An extraterrestrial being is seeking your help in learning how to use the human number system. He/she/it asks you to explain the meaning of the numbers $2\frac{4}{7}$ and 9.637. Draw diagrams to help the extraterrestrial understand our number system.

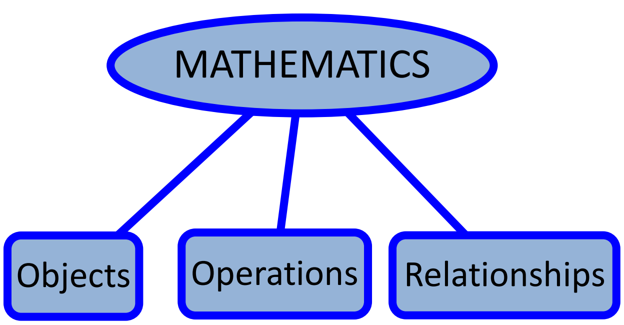


Summary of Important Concepts

The Basic Elements of Mathematics

As shown below, mathematics can be reduced to ***three basic concepts***:

1. Mathematical Objects
   (e.g. ***numbers*** are mathematical objects)
2. Mathematical Operations
   (e.g. +, −, ×, ÷, exponents, $\sqrt{\ }$, etc)
3. Mathematical Relationships
   (e.g. $c^2 = a^2 + b^2$, i.e. the Pythagorean Theorem)



LANGUAGE HAS MEANING!

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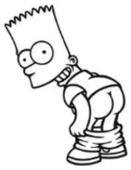
THEREFORE, MATH HAS MEANING!

- Always ask yourself, "What does this mean?"
- Avoid the "autopilot" state of mind!
- +, − Gains and Losses
- × Groups of (Repeated Addition)
- ÷ How many groups of? (Opposite of ×)
- powers Repeated Multiplication

Working with Integers

 always comes last!

Highlight the terms before evaluating the expression!

1 2 3 4

B E DM AS

Division/Multiplication - tied, left-to-right

Addition/Subtraction - tied, left-to-right

Terms are separated by + and - signs.

Separate each expression into terms. Then apply the operations in the correct order.

+(+) add a positive: GAIN

-(-) subtract a negative: GAIN

+(-) add a negative: LOSS

-(+) subtract a positive: LOSS

Gains more than losses: + answer

Losses more than gains: - answer

Rule for Determining the Sign of the Answer when Multiplying and Dividing

Two Numbers

Signs Same: + answer

Signs Different: - answer

More than Two Numbers

Even # of Negatives: + answer

Odd # of Negatives: - answer

Working with Rational Numbers

Multiplying Fractions

- Multiply the numerators, multiply the denominators. If possible, reduce to lowest terms. **OR…**
- Reduce first (vertically and diagonally). Multiply the numerators, multiply the denominators.

$\frac{3}{10}\left(\frac{8}{15}\right)=\frac{24}{150}=\frac{4}{25}$ OR $\frac{\overset{1}{\cancel{3}}}{\underset{5}{\cancel{10}}}\left(\frac{\overset{4}{\cancel{8}}}{\underset{5}{\cancel{15}}}\right)=\frac{4}{25}$

Dividing Fractions

- ***Do not*** change the 1st fraction.
- Change ÷ to ×.
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- **Summary:** Multiply by the reciprocal

$$\frac{3}{10}\div\frac{8}{15}=\frac{3}{10}\times\frac{15}{8}=\frac{45}{80}=\frac{9}{16}$$

Adding/Subtracting Fractions

- Express each fraction with a common denominator.
- Add/subtract the numerators.
- Keep the denominator!
- If possible, reduce to lowest terms.

$$\frac{3}{10}+\frac{8}{15}=\frac{9}{30}+\frac{16}{30}=\frac{25}{30}=\frac{5}{6}$$

Why, Why, Why? You must ask why!

Important Reminder

If you would like the study of mathematics to contribute significantly to your intellectual growth, you ***MUST UNDERSTAND*** the ***MEANING*** of mathematical concepts such as the following:

1. **Numbers**
   e.g. small versus large numbers, negative versus positive numbers, whole numbers versus fractions
2. **Mathematical Operations**
   e.g. "+" means "gain," "–" means "lose," "×" means "groups of," "÷" means "how many groups of"
   "$100 \div \frac{1}{3}$ means "How many groups of $\frac{1}{3}$ can be formed from 100?"
3. **Variables and Expressions**
   e.g. "x" means "an *unknown* or *unspecified* number," "$2x$" means "2 times *any number*"
4. **Mathematical Relationships**
   e.g. "$c^2 = a^2 + b^2$" is an equation that expresses the relationship among the side lengths of a right triangle.

You must always seek to understand *why* something is true. Every claim *requires* a justification!

"You do not really understand something unless you can explain it to your grandmother." (Albert Einstein)

Why on Earth do you do it like that?

1. Why is subtracting a negative number a GAIN? (**e.g.** $5-(-10)=5+10=15$)

   Suppose that you have just borrowed \$100 in cash from a good friend and that you have decided to keep the borrowed money in your wallet. In addition, you have \$500 of your own money, all of which you have deposited into a bank account. Now suppose further that your friend is so charitable that he/she decides that *you do not need to repay the debt*.

   Another way of putting this is that you have *\$500 of your own* and your friend *takes away a debt of \$100*. *Do you gain or lose money?* Write a mathematical expression to support your answer.

2. Why is the product of a positive number and a negative number NEGATIVE? (**e.g.** $5(-10)=-50$)
   Why is the quotient of a positive number and a negative number NEGATIVE? (**e.g.** $-50 \div 5 = -10$)
   Hints: "$5(-10)$" can be interpreted as "five groups of -10." Multiplication and division are *opposites* of each other.

3. Why is the product of two negative numbers POSITIVE? (**e.g.** $-5(-10)=50$)

Why is the quotient of two negative numbers POSITIVE? (**e.g.** $-50\div(-10)=5$)

Hint: Multiplication and division are *opposites* of each other.

4. When multiplying fractions, why do we multiply the numerators and the denominators?

(**e.g.** $\frac{2}{3}\times\frac{5}{7}=\frac{2\times5}{3\times7}=\frac{10}{21}$)

Hint: The operation "multiplication" is equivalent to the operation "of."

5. When dividing fractions, why do we multiply by the reciprocal? (**e.g.** $\frac{2}{3}\div\frac{5}{7}=\frac{2}{3}\times\frac{7}{5}=\frac{14}{15}$)

Hint: Multiplication and division are *opposites* of each other. In addition, for the operations "$\times$" and "$\div$," the *opposite* of a number is its *reciprocal*. (Recall that for the operations "$+$" and "$-$," the *opposite* of a *number* is its negative.)

Developing a Mindset Conducive to *Understanding* Algebra

What you Already Understand + x = Algebra!

What you Already Understand		***What is New***		
• Numbers (**e.g.** integers, rational numbers, irrational numbers) • Operations and their Meanings (**e.g.** +, −, ×, ÷) • Using Numbers and Operations to create Expressions • Applying Operations in the Correct Order (**i.e.** BEDMAS)	+	***Symbols***, usually letters, are used to ***represent*** ("stand for") • ***unknown*** numbers • ***unspecified*** numbers (numbers that are not yet "given")	=	Algebra

Besides using letters to represent unknown numbers,
NOTHING ELSE CHANGES!

Whatever you have already learned about mathematics still applies! All that you must do is get used to working with expressions in which some of the numbers are unknown.

Examples without Unknowns

Addition	Multiplication
$3+3=2(3)=6$	$3(3)=3^2=9$
$4+4=2(4)=8$	$4(4)=4^2=16$
$10+10=2(10)=20$	$10(10)=10^2=100$
$3+3+3+3=4(3)=12$	$3(3)(3)(3)=3^4=81$
$5+5+5+5=4(5)=20$	$5(5)(5)(5)=5^4=525$
$10+10+10+10=4(10)=40$	$10(10)(10)(10)=10^4=10000$

Examples with Unknowns

Addition	Multiplication
$x+x=2x$	$x(x)=x^2$
$y+y=2y$	$y(y)=y^2$
$z+z=2z$	$z(z)=z^2$
$x+x+x+x=4x$	$x(x)(x)(x)=x^4$
$y+y+y+y=4y$	$y(y)(y)(y)=y^4$
$z+z+z+z=4z$	$z(z)(z)(z)=z^4$

Exercises

1. Use the examples on the previous page as a model to ***simplify*** (write in the simplest possible form) each of the following algebraic expressions:

Addition	***Multiplication***
(a) $x+x$	**(b)** $x(x)$
(c) $y+y+y$	**(d)** $y(y)(y)$
(e) $z+z+z+z+z$	**(f)** $z(z)(z)(z)(z)$
(g) $2x+2x+2x+2x+2x+2x$	**(h)** $2x(2x)(2x)(2x)(2x)(2x)$
(i) $5y+5y+5y+5y$	**(j)** $5y(5y)(5y)(5y)$
(k) $z+2z+3z+4z$	**(l)** $z(2z)(3z)(4z)$
(m) $z+2z+3z-4z$	**(n)** $z(2z)(3z)(-4z)$
(o) $a+2b+3a-4b$	**(p)** $a(2b)(3a)(-4b)$
(q) $-a-2b+3a-4b$	**(r)** $-a(-2b)(3a)(-4b)$
(s) $-a-2b+3c-4d$	**(t)** $-a(-2b)(3c)(-4d)$

2. A mathematically naïve classmate insists that $x+x$ simplifies to x^2. Use a simple counterexample to disprove your classmate's claim. (A ***counterexample*** is an example that shows that a statement is false.)

Answers

Question 1

(a) $2x$

(b) x^2

(c) $3y$

(d) y^3

(e) $5z$

(f) z^5

(g) $12x$

(h) $2^6x^6=64x^6$

(i) $20y$

(j) $5^4y^4=625y^4$

(k) $10z$

(l) $2(3)(4)z^4=24z^4$

(m) $2z$

(n) $2(3)(-4)z^4=-24z^4$

(o) $4a-2b$

(p) $2(3)(-4)a^2b^2=-24a^2b^2$

(q) $2a-6b$

(r) $-1(-2)(3)(-4)a^2b^2=-24a^2b^2$

(s) Cannot be simplified

(t) $-1(-2)(3)(-4)abcd=-24abcd$

Question 2

If the expressions $x+x$ and x^2 are indeed equal, the two expressions must have the same value no matter what the value of x is. Consider what happens when $x = 1$:

$$x+x=1+1=2$$
$$x^2=1^2=1(1)=1$$

Since the expressions have different values when $x = 1$, the expressions cannot be equal.

Also, $x+x$ means "a number ***added*** to itself" whereas x^2 means "a number ***multiplied*** by itself," which clearly are not the same.

THE LANGUAGE OF ALGEBRA

Letters Mixing with Numbers? What on Earth is Going On?

After many years of securely navigating the concrete world of the digits 0 through 9, the dreaded first algebra lesson inevitably arrives. Suddenly, a dizzying array of letters and other symbols swoop upon math students, often leaving them feeling as if a thick, obscuring fog had instantaneously materialized in their mathematical world. There is no need to despair, however. With a consistent focus on ***meaning***, a positive ***attitude*** and a lot of ***effort***, the fog will eventually lift and clarity will be restored.

Definition of Algebra: the branch of mathematics in which letters and other symbols are used to represent numbers and other quantities in expressions and equations.

Examples

Algebra is a Language? Quantitative versus Qualitative Description

Quantitative: relating to, measuring, or measured by the ***quantity*** of something rather than its quality.

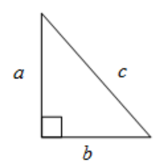
Qualitative: relating to, measuring, or measured by the ***quality*** of something rather than its quantity.

Because it is not widely known that mathematical symbols can be used to express ideas, students are often surprised to learn that mathematics involves language. Admittedly, the language of math fails miserably (at least for the foreseeable future) in describing the beauty of a work of art, the feelings evoked by a haunting passage of music or the joy of being reunited with a loved one. Clearly, natural languages like English are far better suited to descriptions of a ***qualitative*** nature. When considering descriptions of a ***quantitative*** nature, however, math is decidedly the victor. The examples given below point out the advantages of using the language of mathematics in quantitative investigations.

Example 1 – The Pythagorean Theorem

The Pythagorean Theorem describes a relationship that exists among the side lengths of a right triangle. The table below shows how this theorem can be described using both English and the language of algebra.

English
In ***any*** right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.



Algebra
In ***any*** right triangle, $c^2 = a^2 + b^2$

As outlined below, a number of difficulties arise when English is used to describe mathematical relationships.

1. The wording tends to be long and cumbersome, which can easily cause confusion. Even a relationship as simple as the Pythagorean Theorem is quite difficult to describe in English.
2. In the absence of a translation into other languages, the description is accessible only to those who have a reading knowledge of English.
3. It is extremely difficult to manipulate relationships expressed in English or any other natural language.

By using an algebraic approach, however, these difficulties can be overcome quite easily.

1. The use of algebraic symbols results in very concise descriptions of relationships.
2. Algebraic descriptions are accessible to all people who have a rudimentary understanding of mathematics. Ethnic background is irrelevant.
3. Relationships can be manipulated very easily as shown in the following example:

$$c^2 = a^2 + b^2$$
$$\therefore b^2 = c^2 - a^2$$
$$\therefore a^2 = c^2 - b^2$$

Example 2 – Using Equations to Solve Problems

There are one hundred and forty coins in a collection of dimes and nickels. If the total value of the coins is \$10.90, how many dimes and how many nickels are there?

Solution

The main difficulty encountered in solving this type of problem is the ***translation*** from English into the language of algebra. Once this is done, the relationship between the number of coins and the total value of the coins is expressed in a form that is easy to manipulate. The following approach is usually not found in textbooks but it clearly shows how the equation that is obtained is nothing more than a ***mathematical restatement of the English description of the problem***.

The total value of the coins **is** \$10.90.

The total value of the coins = \$10.90

The value of all the nickels + the value of all the dimes = \$10.90

0.05×(the number of nickels) + 0.10×(the number of dimes) = \$10.90

Now we are ready to apply the language of algebra. If n represents the number of nickels, then the number of dimes must be equal to $140 - n$ (since there are 140 coins altogether). Finally, it is possible to write the following mathematical representation of the original English description:

$$0.05n + 0.10(140 - n) = \$10.90$$

By solving this equation, the desired result is obtained.

Vocabulary of Algebra

Important Definitions

Constant
Any ***fixed*** value or any expression that evaluates to a fixed value.

e.g. 4, 62, –4573, π

$$-3+5(-3)^2-5(3-4(5))$$

Variable
A ***symbol***, usually a ***letter***, which represents an ***unknown*** or ***unspecified*** value. As the name implies, a variable is a quantity that ***can change*** or that may take on ***different values***.

e.g. x, y, a, z, θ, Δ, α, β, ☺, ☹, □

(Algebraic) Expression
Any mathematical calculation combining constants and/or variables using any valid mathematical operations.

e.g. $-3x^2y+5abc-\dfrac{2xy^3z}{ab^2}-5\sqrt{z}$

Term
Parts of an expression separated by ***addition*** and ***subtraction*** symbols. More precisely, a term is any mathematical calculation combining constants and/or variables using any operations ***except for addition and subtraction***.

e.g. In the ***expression***

$$-3x^2y+5abc-\frac{2xy^3z}{ab^2}-5\sqrt{z},$$

the ***terms*** are $-3x^2y$, $5abc$, $-\dfrac{2xy^3z}{ab^2}$ and $-5\sqrt{z}$.

(Numeric) Coefficient
The ***constant*** part of a term.

e.g. In the term $-3x^2y$, the ***numeric coefficient*** (or just coefficient) is −3.

Literal Coefficient (Variable Part)
The ***variable*** part of a term.

e.g. In the term $-3x^2y$, the ***literal coefficient*** is x^2y.

Like and Unlike Terms
Two terms with exactly the same literal coefficients are called ***like terms***.

e.g. $-3x^2y$ and $16x^2y$ are ***like terms***. On the other hand, $-3x^2y$ and $16xy$ are ***unlike terms***.

Evaluate an Expression
To calculate or compute with the objective of finding the “final answer.”

e.g. The expression $12+3\sqrt{25}$ ***evaluates*** to 27.

Simplify an Expression
Use the rules of algebra and arithmetic to write an expression in the simplest possible form.

e.g. The expression $2a+5a$ ***simplifies*** to . (Two apples plus five apples is seven apples.) On the other hand, $2a+5b$ ***cannot be simplified*** because $2a$ and $5b$ are ***unlike terms***. (Two apples plus five bananas ***is not equal*** to “seven apple-bananas.”)

Polynomial
A ***polynomial expression*** is an expression in which ***each term*** consists of constants and/or variables combined using ***only multiplication*** (including powers of the variables).

e.g. 1, −3, $-3x^2y$, $2ab-6a^2bc$, x^2+2x+1, x^3+3x^2+3x+1

Note that although the prefix “poly” means “many” or “multiple,” the term polynomial can be used to describe such an expression with ***any number of terms***.

Note: It is important to remember that ***no operations other than +, − and ×*** can be performed to the variables in polynomials.

e.g. $-3x^2y$ ✓ $\dfrac{-3x^2}{y}$ ✗ $\sqrt{\dfrac{-3x^2}{y}}$ ✗

Monomial
A polynomial with ***exactly one term***.

e.g. Three examples of monomials: 1, −3, $-3x^2y$

Binomial
A polynomial with ***exactly two terms***.

e.g. $2ab-6a^2bc$

Trinomial
A polynomial with ***exactly three terms***.

e.g. x^2+2x+1

n-Term Polynomial
For the sake of not having to remember a large number of names, polynomials with more than 3 terms are usually called “*n*-term polynomials.”

e.g. $-5x^7+3x^4-5x^2+2x+1$ is a ***5-term polynomial***

Degree of a Polynomial
In any term of a polynomial, the ***degree of the term*** is found by adding the exponents of all the variables. The degree of the polynomial is the ***degree of the highest-degree term***.

e.g. $-4x^3y-3x^2y^2+3xy^4+1$

Term	Degree
$-4x^3y$	3 + 1 = 4
$-3x^2y^2$	2 + 2 = 4
$3xy^4$	1 + 4 = **5**
1	0

Therefore, the degree of the polynomial is 5.

Equivalent Expressions
Two expressions are ***equivalent*** if they can be simplified to exactly the same expression. For example, $2a+5a$ and $7a$ are equivalent because both expressions simplify to $7a$. In addition, equivalent expressions must agree for all possible values of the variable(s). **e.g.** $(x+y)^2$ is equivalent to $x^2+2xy+y^2$ because the expressions agree ***for all values*** of x and y.

Expressions and Equations – Mathematical Phrases and Sentences

- ***equation*** → L.H.S. = R.H.S. → a complete mathematical "***sentence***"
  e.g. "The sum of two consecutive numbers is 31." → $x+x+1=31$
- ***expression*** → ***not*** a complete mathematical "sentence" → more like a ***phrase***
  e.g. "Ten more than a number" → $x+10$

Solving the so-called "word problems" that you are given in school is usually just a matter of ***translating English sentences into mathematical equations***.

Mathematical Words

Symbol	*English Equivalent*
+	**sum**, plus, added to, more than, increased by, gain of, total of, combined with
–	**difference**, minus, subtracted from, less than, fewer than, decreased by, loss of
×	**product**, times, multiplied by, of, factor of, double (×2), twice (×2), triple (×3)
÷	**quotient**, divided by, half of (÷2), one-third of (÷3), per, ratio of
=	**is**, are, was, were, will be, gives, yields

Translating from English into Algebraic Expressions and Equations

Complete the following table.

English	*Algebraic Expression*	*English*	*Algebraic Equation*
Six more than a number	$n+6$	Six more than a number is 5.	$n+6=5$
A number decreased by 7	$x-7$	A number decreased by 7 is –9.	$x-7=-9$
The product of a number and –3	$-3y$	The product of a number and –3 is 4.	$-3y=4$
Half of a number	$\frac{z}{2}$	Half of a number is 16.	$\frac{z}{2}=16$
Triple a number decreased by 5	$3x-5$	Triple a number decreased by 5 is 8.	$3x-5=8$
Double a number plus 5		Double a number plus 5 gives 13.	
One-third of a number minus 2		One-third of a number minus 2 yields 16.	
	$\frac{x}{4}-5$		$\frac{x}{4}-5=-1$
Sixty-five decreased by a number		Sixty-five decreased by a number gives 7.	
A number divided by 7		A number divided by 7 is –10.	
Quadruple a number subtracted ***from*** 6		Quadruple a number subtracted ***from*** 6 is 2.	
	$2-4t$		$2-4t=3$
	$\frac{2}{x-4}$		$\frac{2}{x-4}=-9$
The quotient of 6, and a number subtracted from 3		The quotient of 6 and a number subtracted from 3 is 2.	
The product of 2, and a number increased by 7		The product of 2, and a number increased by 7 is 13.	
The difference of triple a number, and a number increased by 3		The difference of triple a number, and a number increased by 3 yields 21.	

Practice: Communicate With Algebra

1. For each term, identify the coefficient (numeric coefficient) and the variable part (literal coefficient).
   a) $4x$ **b)** $-5p^4$
   c) $3m^2n$ **d)** g^3h^2
   e) $-2y^5$ **f)** $-p^4q^5$
   g) $\frac{3}{4}ab$ **h)** $0.6r^4s^2$

2. The expression $2x + 5$ is a:
   A monomial **B** binomial
   C trinomial **D** term

3. The expression $-12m^4n$ is a:
   A monomial **B** binomial
   C trinomial **D** term

4. The expression $3a^2b^2 + ab^3 + b$ is a:
   A monomial **B** binomial
   C trinomial **D** term

5. Classify each polynomial by type.
   a) $2x + 1$
   b) $3p^2 - p + 4$
   c) $4b^2d^3$
   d) $6 + gh^5$
   e) $2 - y^5 - y^2 + 4y$
   f) $x^2 - y^2 + 4$
   g) $ab - b$
   h) $6p^3q^3$

6. What is the degree of each term in question 5?

7. The degree of $5m^2n + mn^3 + 1$ is:
   A 1 **B** 2
   C 3 **D** 4

8. What is the degree of each polynomial?
   a) $6a^2 + 4b^3$ **b)** $5b^4$
   c) $3x^2 + x - 1$ **d)** $m^3 - m^2 + 4m$
   e) $2p^4q^3$ **f)** $x^2y^2 + 4xy$
   g) $a^5b - 7b^3$ **h)** $-m^4n^3 - m^2n + 4mn^4$

9. Which equation matches this phrase: a number increased by 6 is 8
   A $6x = 8$ **B** $x + 8 = 6$
   C $x + 6 = 8$ **D** $\frac{x}{6} = 8$

10. Write an equation for each phrase.
    a) double a number is 14
    b) a number decreased by 6 is 5
    c) one third of a number is 2
    d) triple a number, increased by 1 is 8

11. Maggie earns \$5 per hour when she babysits 1 child. She earns \$8 per hour when she babysits 4 children. Let x represent the number of hours she babysits one child and y represent the number of hours she babysits four children. Which expression represents her total earnings?
    A $5x - 8y$ **B** $x + y$
    C $5x + 8y$ **D** $x - y$

12. Evaluate each expression for the given values of the variables.
    a) $2x - 3$ $x = 4$
    b) $3y + 2$ $y = 7$
    c) $r^2 - r + 1$ $r = 6$
    d) $a^2 - 2b^2$ $a = 3, b = 1$
    e) $p^2 + 2p - 3$ $p = 4$
    f) $4x^2 - y - 2$ $x = 2, y = 1$

Answers

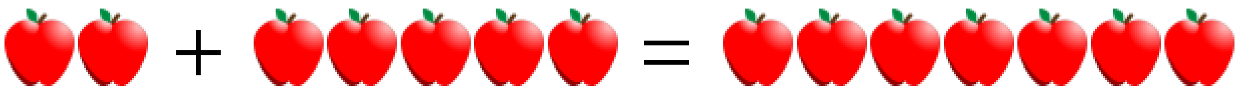
1. **a)** coefficient: 4; variable: x
   b) coefficient: –5; variable: p^4
   c) coefficient: 3; variable: m^2n
   d) coefficient: 1; variable: g^3h^2
   e) coefficient: –2; variable: y^5
   f) coefficient: –1; variable: p^4q^5
   g) coefficient: $\frac{3}{4}$; variable: ab
   h) coefficient: 0.6; variable: r^4s^2
2. **B:** binomial
3. **A:** monomial and **D:** term
4. **C:** trinomial
5. **a)** binomial **b)** trinomial
   c) monomial **d)** binomial
   e) four-term polynomial
   f) trinomial
   g) binomial **h)** monomial
6. **a)** 1 **b)** 2 **c)** 5 **d)** 6
   e) 5 **f)** 2 **g)** 2 **h)** 6
7. **D:** 4
8. **a)** 3 **b)** 4 **c)** 2 **d)** 3
   e) 7 **f)** 4 **g)** 6 **h)** 7
9. C
10. **a)** $2x = 14$ **b)** $x - 6 = 5$
    c) $\frac{x}{3} = 2$ **d)** $3x + 1 = 8$
11. C
12. **a)** 5 **b)** 23 **c)** 31
    d) 7 **e)** 21 **f)** 13

SIMPLIFYING ALGEBRAIC EXPRESSIONS INVOLVING ADDITION AND SUBTRACTION

Definition of "Simplify"

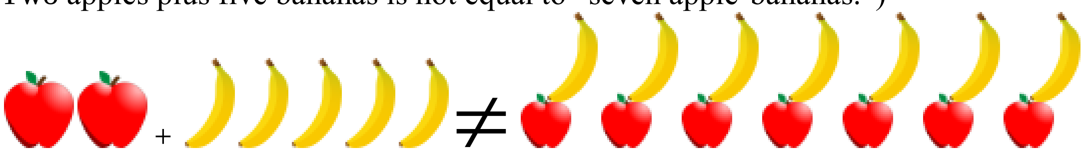
Use the rules of algebra and arithmetic to write an expression in the simplest possible form.

e.g. The expression $2a+5a$ ***simplifies*** to $7a$. (Two groups of *a* plus five groups of *a* must be seven groups of *a*. Two apples plus five apples ***is equal to*** seven apples.)



On the other hand, $2a+5b$ cannot be simplified because $2a$ and $5b$ are unlike terms.

(There is no way to make "two groups of *a* plus five groups of *b*" any simpler.
Two apples plus five bananas is not equal to "seven apple-bananas.")



Important Points to Remember when Simplifying Polynomials that don't Contain Brackets

- Keep in mind and apply correctly the rules for adding and subtracting integers. The main idea underlying addition and subtraction of integers is that these operations all boil down to either a ***loss*** (move down, move left, etc) or a ***gain*** (move up, move right, etc)

  + (+) = + = ***add*** a ***positive*** value = ***gain*** — – (–) = + = ***subtract*** a ***negative*** value = ***gain***

  +(–) = – = ***add*** a ***negative*** value = ***loss*** — – (+) = – = ***subtract*** a ***positive*** value = ***loss***

- Remember to look for ***like terms***. Do not fall into the trap of attempting to simplify the sum or difference of ***unlike terms***. (2 cows + 2 cows = 4 cows   2 boys + 2 boys = 4 boys   2 cows + 2 boys ≠ 4 cowboys)

Simplifying Expressions involving Two or More Terms and NO Brackets

Examples

Simplify each of the following polynomials:

<table>
<tr>
<td>1.
$-5a+3a-6b+4b$
$=-2a-2b$</td>
<td>2.
$-2x^2y-7x^2y+3xy-8xy$
$=-9x^2y-5xy$
Note that x^2y and xy are NOT like terms. The term x^2y means xxy, which is different from xy.</td>
<td>3.
$-5a-6b+3a-4b$
$=-5a+3a-6b-4b$
$=-2a-10b$
Collect Like Terms
The operations must not change! (e.g. +3a remains +3a and −6b remains −6b)</td>
</tr>
<tr>
<td colspan="2">4.
$-5a+6b+3c-4d$
This polynomial cannot be simplified because there are no like terms. (In your notes or on a test, you may write "CBS" as a short form for "cannot be simplified.")</td>
<td>5.
$-15ab^2-6a^2b-13ab^2-4a^2b-10ab$</td>
</tr>
</table>

Practice: Collect Like Terms

1. Which polynomial contains a term like xy^2?

A $4xy - x^2y$ **B** $2x^2 + 3xy^2$

C $-x + y^2 - xy$ **D** $x^2 + y^2 + 4$

2. Are the terms in each pair like or unlike?

a) $5a$ and $-2a$

b) $3x^2$ and x^3

c) $2p^3$ and $-p^3$

d) $4ab$ and $\frac{2}{3}ab$

e) $-3b^4$ and $-4b^3$

f) $6a^2b$ and $3a^2b$

g) $9pq^3$ and $-p^3q$

h) $2x^2y$ and $3x^2y^2$

3. Write one like term and one unlike term for each.

a) $4p$ **b)** $-3a^2$

c) $-k^3$ **d)** $2x$

e) $-4mn^4$ **f)** $2ab$

g) $-pq^3$ **h)** $3b^2d^2$

4. Is it possible to simplify each expression? How do you know?

a) $8a + 3a$ **b)** $5m + 2n$

c) $3p + p$ **d)** $3t - 7t$

e) $4x - 3$ **f)** $-v - 4v + 2v$

g) $6c^2 - c^2 - 3c^2$ **h)** $r^2 + 3r + 7$

5. Simplify each expression.

a) $p + 2p$ **b)** $7g - 4g$

c) $2a - 8a$ **d)** $5x - 2x$

e) $6q + q$ **f)** $4y^2 + 5y^2$

g) $u + 4u - u$ **h)** $7b^3 - 2b^3 - b^3$

6. Collect like terms. Then, simplify.

a) $4b + 3 - 2b + 1$

b) $2p - 7 - p + 4$

c) $1 + 3y + 4 + y$

d) $5 - x - 1 - 2x$

e) $6a - 2b + 3b + 2a$

f) $7r + 2 + 3r - r - 1$

g) $9s - 2s + 5t - 4s$

h) $-g - 3h + 5h + 2g - h$

7. Simplify.

a) $4 + v + 5v - 10$

b) $7a - 2b - a - 3b$

c) $8k + 1 + 3k - 5k + 4 + k$

d) $2x^2 - 4x + 8x^2 + 5x$

e) $12 - 4m^2 - 8 - m^2 + 2m^2$

f) $-6y + 4y + 10 - 2y - 6 - y$

g) $5 + 3h + h - 4 + h + 6 + 2h$

h) $4p^2 + 2q^2 - p^2 + 3p^2 - 7q^2$

8. Simplify.

a) $2a + 6b - 2 + b - 4 + a$

b) $4x + 3xy + y + 5x - 2xy - 3y$

c) $m^4 - m^2 + 1 + 3 - 2m^2 + m^4$

d) $x^2 + 3xy + 2y^2 - x^2 + 2xy - y^2$

9. The length of a rectangle is 2 times the width of the rectangle. Let x represent the width of the rectangle.

a) Write an expression to represent the length of the rectangle.

b) Write a simplified expression for the perimeter of the rectangle.

c) Suppose the width is 6 cm. Find the perimeter of the rectangle.

Answers

1. **B:** $2x^2 + 3xy^2$

2. **a)** like **b)** unlike **c)** like **d)** like **e)** unlike **f)** like **g)** unlike **h)** unlike

3. **a)** like: $-7p$; unlike: $4x$
b) like: $2a^2$; unlike: $-3a$
c) like: $5k^3$; unlike: $-k^2$
d) like: $-3x$; unlike: $4p$
e) like: mn^4; unlike: m^4n
f) like: $4ab$; unlike: $4a$
g) like: $2pq^3$; unlike: p^2q^3
h) like: $-b^2d^2$; unlike: $3bd$

4. **a)** yes; both terms have the variable a
b) no; the terms have different variables
c) yes; both terms have the variable p
d) yes; both terms have the variable t
e) no; the terms have different variables
f) yes; all terms have the variable v
g) yes; all terms have the variable c^2
h) no; the terms do not all have the same variables

5. **a)** $3p$ **b)** $3g$ **c)** $-6a$ **d)** $3x$ **e)** $7q$ **f)** $9y^2$ **g)** $4u$ **h)** $4b^3$

6. **a)** $4b - 2b + 3 + 1$; $2b + 4$
b) $2p - p - 7 + 4$; $p - 3$
c) $1 + 4 + 3y + y$; $5 + 4y$
d) $5 - 1 - x - 2x$; $4 - 3x$
e) $6a + 2a - 2b + 3b$; $8a + b$
f) $7r + 3r - r + 2 - 1$; $9r + 1$
g) $9s - 2s - 4s + 5t$; $3s + 5t$
h) $-g + 2g - 3h + 5h - h$; $g + h$

7. **a)** $6v - 6$ **b)** $6a - 5b$ **c)** $7k + 5$ **d)** $10x^2 + x$ **e)** $4 - 3m^2$ **f)** $-5y + 4$ **g)** $7 + 7h$ **h)** $6p^2 - 5q^2$

8. **a)** $3a + 7b - 6$ **b)** $9x + xy - 2y$ **c)** $2m^4 - 3m^2 + 4$ **d)** $5xy + y^2$

9. **a)** $L = 2x$ **b)** $P = 6x$ **c)** 36cm

Simplifying Expressions involving Two or More Terms AND Brackets

Be Careful!

Be careful when simplifying expressions that are enclosed in brackets. The brackets can be removed without any changes ***only when a polynomial is being added***. If a polynomial enclosed in brackets is being subtracted, you must bear in mind that the ***operation of subtraction applies to the entire polynomial***, not just the first term.

- Addition ***can*** be performed in any order without changing the result. This means that brackets don't matter! Therefore, to ***add a polynomial*** enclosed in brackets, ***simply remove the brackets*** and proceed.
- Subtraction ***cannot*** be performed in any order without changing the result. This means that brackets ***DO*** matter! To ***subtract a polynomial*** enclosed in brackets, remove the brackets by ***adding the opposite of the polynomial***. This is based on the following property: $x - y = x + (-y)$

Why this Works

- We already know that $x - y = x + (-y)$.
- In other words, ***subtraction*** is the same as "adding the negative of."
- "Adding the negative of" is also the same as "adding the ***opposite*** of."
- Therefore, ***subtraction*** is the same as "adding the ***opposite*** of."

Evaluate each expression and then draw conclusions.

(a) $2-3+5$

(b) $2-(3+5)$

(c) $2+(-3-5)$

Conclusions

Examples

1.

$$(-5a+6b)+(3a-4b)$$
$$=-5a+6b+3a-4b$$
$$=-5a+3a+6b-4b$$
$$=-2a+2b$$

Since a polynomial is being *added*, the brackets don't matter. They can be removed without making any changes because the result of adding numbers is unaffected by the order in which the numbers are added. That is,

a+b = b+a

and

a+(b+c) = (a+b)+c

These are called respectively the *commutative* and *associative properties* of addition.

2.

$$(-5a+6b)-(3a-4b)$$
$$=-5a+6b+(-3a+4b)$$
$$=-5a+6b+(-3a)+4b$$
$$=-5a+6b-3a+4b$$
$$=-5a-3a+6b+4b$$
$$=-8a+10b$$

Since a polynomial is being *subtracted*, the brackets DO matter.

Subtraction is *neither* commutative *nor* associative.

Add the Opposite because subtracting a number is equivalent to adding the *negative* or *opposite* of the number.

3.

$$(-5x+1)-(2x-7)-(-3x+5)$$
$$=-5x+1+(-2x+7)+(3x-5)$$
$$=-5x+1+(-2x)+7+3x-5$$
$$=-5x+1-2x+7+3x-5$$
$$=-5x-2x+3x+1+7-5$$
$$=-4x+3$$

Two of the brackets are preceded by a subtraction sign.
In each case, ***add the opposite***:

$$-(2x-7)\rightarrow+(-2x+7)$$
$$-(-3x+5)\rightarrow+(3x-5)$$

The leftmost set of brackets is not preceded by a negative sign. The brackets can be removed without making any changes.

$$(-5x+1)\rightarrow-5x+1$$

Practice: Add and Subtract Polynomials

1. Which expression represents the result of simplifying $(3x - 4) + (2x + 1)$?

A $6x - 4$ **B** $6x + 4$
C $5x + 3$ **D** $5x - 3$

2. Write without brackets and collect like terms. Then, simplify.

a) $(x + 3) + (x + 5)$
b) $(2y - 5) + (y + 9)$
c) $(5m + 1) + (2m + 2)$
d) $(3 - 4d) + (d - 1)$
e) $(3v - 2) + (6 - v)$
f) $(k + 4) + (2 - 3k) + (6k - 1)$
g) $(2p + 4) + (p - 2) + (8 - 3p)$
h) $(3 - r) + (4 + 5r) + (2r - 1)$

3. Write the opposite of each expression.

a) 3 **b)** -5
c) $2p$ **d)** $-x$
e) $m + 4$ **f)** $3b - 1$
g) $x^2 + 2x - 4$ **h)** $-6 - y^2$

4. Which expression represents the result of simplifying $(4x - 1) - (x + 1)$?

A $5x + 2$ **B** $3x - 2$
C $5x$ **D** $3x$

5. Add the opposite. Then, simplify.

a) $(4d - 2) - (d + 1)$
b) $(3x - 4) - (x + 3)$
c) $(2p + 5) - (3p + 2)$
d) $(8 - 4m) - (m - 2)$
e) $(a - 2) - (5 - 3a)$
f) $(z + 7) - (4 - z)$
g) $(p + 1) - (p - 2)$
h) $(5 - 2b) - (6 + 4b)$

6. Simplify.

a) $(6k - 4) + (2k + 4)$
b) $(n + 3) + (3n - 5)$
c) $(2a + 1) - (4a + 2)$
d) $(5 - 3m) + (2m - 1)$
e) $(b - 6) - (2 - 5b) + (b + 4)$
f) $(x + 2) - (1 - x) - (5 + x)$
g) $(g + 12) + (g - 7) - (2 - 3g)$
h) $(1 - b) + (3 + 2b) - (b - 8)$

7. Simplify.

a) $(x^2 + 2x + 1) + (2x^2 + 4)$
b) $(4a + 3b - 6) + (2a - b + 4)$
c) $(2m^2 + m + 12) + (3m^2 + 4m - 6)$
d) $(5n + mn - 3m) + (2m - 5mn + n)$

8. A rectangle has length $4x + 1$ and width $x + 2$.

a) Write a simplified expression for the perimeter of the rectangle.
b) Find the perimeter of the rectangle when $x = 5$.

9. Three artists contributed to a coffee-table book. They each chose to be paid a different way.

Artist	Fixed Rate ($)	Royalty ($ per n books sold)
Ayesha	1000	$2n$
Jorge	–	$5n$
Ioana	4000	–

a) Write an expression for the total earnings for each artist.
b) Write a simplified expression for the total amount paid to Ayesha, Jorge, and Ioana.

Answers

1. **D:** $5x - 3$

2. **a)** $2x + 8$ **b)** $3y + 4$
c) $7m + 3$ **d)** $2 - 3d$
e) $2v + 4$ **f)** $4k + 5$
g) 10 **h)** $6 + 6r$

3. **a)** -3 **b)** 5
c) $-2p$ **d)** x
e) $-m - 4$ **f)** $-3b + 1$
g) $-x^2 - 2x + 4$ **h)** $6 + y^2$

4. **B:** $3x - 2$

5. **a)** $3d - 3$ **b)** $2x - 7$
c) $-p + 3$ **d)** $10 - 5m$
e) $4a - 7$ **f)** $2z + 3$
g) 3 **h)** $-1 - 6b$

6. **a)** $8k$ **b)** $4n - 2$
c) $-2a - 1$ **d)** $4 - m$
e) $7b - 4$ **f)** $x - 4$
g) $5g + 3$ **h)** 12

7. **a)** $3x^2 + 2x + 5$ **b)** $6a + 2b - 2$
c) $5m^2 + 5m + 6$ **d)** $6n - 4mn - m$

8. **a)** $P = 10x + 6$ **b)** 56

9. **a)** *Ayesha*: $1000 + 2n$; Jorge: $5n$; Ioana: 4000
b) *Total* $= 5000 + 7n$

Summary of Main Ideas

Algebra as a Language

Complete the following statements:

(a) Languages like English are best suited to descriptions of a ______________________________ nature.

(b) The language of algebra is best suited to descriptions of a ______________________________ nature.

(c) Math is like a dating service because it's all about ____.

(d) The language of algebra has many advantages when it comes to describing mathematical relationships. Some of the advantages include ____

____

(e) Expressions and equations can be compared to phrases and sentences respectively.
Give an example of an expression: _______________ Give an example of an equation: _________________
An expression is like a phrase because ____.
An equation is like a sentence because ____.

(f) The Pythagorean Theorem is an example of an ______________ that describes the mathematical __________________ among the side lengths of a _____________________________.

(g) Math is much easier to understand when we keep in mind the ______________________ of the symbols, operations, expressions and equations. Also, it helps to have good control over one's mental ____________.

Vocabulary of Algebra

Complete the following table:

Name	***Example***	***Name***	***Example***
Constant			$3x$, $3y$
Variable			$3ab^2$, $-10ab^2$
Expression			$3ab^2 - 10ab^2 = -7ab^2$
Term			$3(-3)(-4)^2 - 10(3)(4)^2 = -304$
(Numeric) Coefficient			$-4x^5 + 2x^3 - 7x^2 + x - 1$
Literal Coefficient (Variable Part)			$-4x^5 + 2x^3 - 7x^2$
Polynomial			$3ab^2 - 10ab^2$
Monomial			$-7x^2$
Binomial			$-3x^2y + 5abc - \frac{2xy^3z}{ab^2} - 5\sqrt{z}$
Trinomial			$-5\sqrt{z}$
Evaluate an Expression			$-5\sqrt{z}$
Simplify an Expression			$-5\sqrt{z}$
Like Terms			a
Unlike Terms			-34553476348.467674737

Simplifying Algebraic Expressions

1. Complete the following statements:

(a) "Evaluate an expression" means ____.

(b) "Simplify an expression" means ____.

(c) When $-3(-3)-10(-3)(5)^2$ is evaluated, the result is ____.

(d) The expression $3ab^2-10ab^2$ ***can*** be simplified because _______________________________________.

(e) The expression $3ab-10ab^2$ ***cannot*** be simplified because _____________________________________.

(f) One way to interpret the expression $2p+5p$ is two ________________ plus five _________________.

Using this interpretation, it ***makes sense*** that the simplified form is $7p$ because

____.

(g) One way to interpret the expression $2h+5d$ is two ________________ plus five _________________.

Using this interpretation, it ***does not make sense*** that the simplified form is $\cancel{7hd}$ because ____________

____.

(h) When simplifying expressions, first ________________________ should be _________________.

When this is being done, it is very important that the _______________ move with the ______________.

(i) When simplifying expressions containing brackets, the brackets can be removed without making any other changes only if the bracket is preceded by a ___________ sign. This is so because ____________ can be performed in any order whatsoever without affecting the sum. If a bracket is preceded by a ___________ sign, brackets cannot be removed without making other changes. This is so because the result of _______________ (i.e. the difference) ***is*** affected by the order in which it is performed. In this case, it is best to write the expression without brackets by ___________________________________.

2. Simplify each of the following expressions.

a) $(7x-9)+(x-4)$

b) $(3y+8)+(-y-5)$

c) $(8c-6)-(c+7)$

d) $(k+2)-(3k-2)$

e) $(3p^2-8p+1)+(9p^2+4p-1)$

f) $(5xy^2+6x-7y)-(3xy^2-6x+7y)$

g) $(4x-3)+(x+8)-(2x-5)$

h) $(2uv^2-3v)-(v+3u)+(4uv^2-9u)$

Answers to Question 2

(a) $8x-13$
(b) $2y+3$
(c) $7c-13$
(d) $-2k+4$
(e) $12p^2-4p$
(f) $2xy^2+12x-14y$
(g) $3x+10$
(h) $6uv^2-12u-4v$

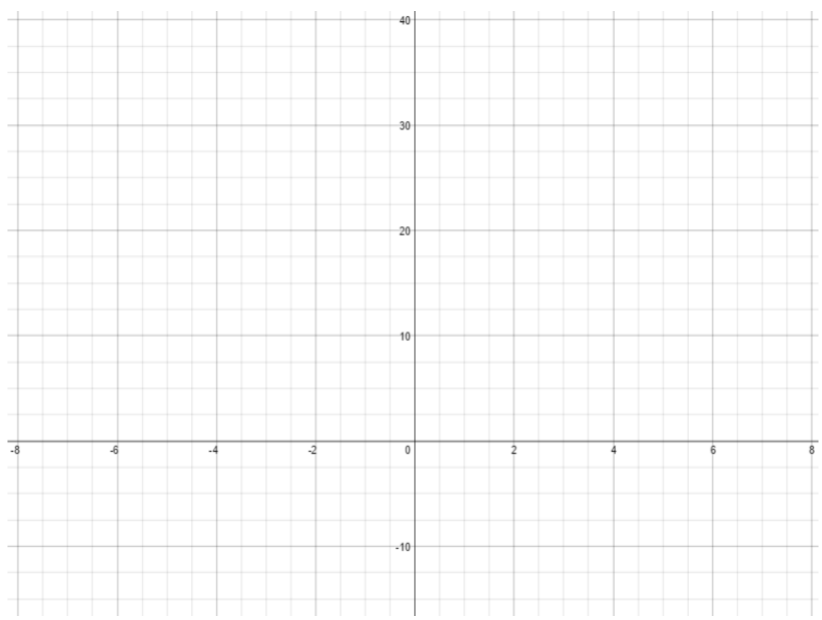
Identifying Polynomials

1. Complete the following table. For each expression that is a polynomial, state its degree and explain how you determined it. Otherwise, explain why it is not a polynomial.

Algebraic Expression	***Is it a Polynomial?***	***Degree (If applicable)***	***Explanation***
$\frac{5}{4}x^2y-\frac{1}{2}xy+3$			
$\frac{5x^2}{4y}-\frac{1}{2}xy+3$			
$-x^2y-\frac{1}{2}x\sqrt{y+3}$			
$-2^xy-\frac{1}{2}x(y+3)$			

2. Use the online graphing calculator "Desmos" (https://www.desmos.com/calculator) to answer the following question:

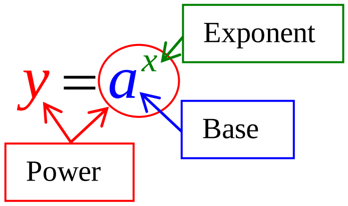
(a) Use the provided grid to sketch the graph of each of the following mathematical relationships. In addition, indicate whether the given relationship is a polynomial.

Equation	***Polynomial?***	***Graphs***
$y=x^2$	Yes / No	
$y=x^3-9x$	Yes / No	
$y=x^4-8x^2+16$	Yes / No	
$y=2^x$	Yes / No	
$y=\frac{1}{x}$	Yes / No	

(b) Describe how the behaviour of the polynomial relationships differs from that of the relationships that are not polynomials.

Simplifying Algebraic Expressions Involving Multiplication and Division

Powers and Laws of Exponents



Meaning of Powers

Powers are a short form for ***repeated multiplication***.

e.g. $4^6 = (4)(4)(4)(4)(4)(4) = 4096$, $\left(-\frac{2}{3}\right)^5 = \left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right)\left(-\frac{2}{3}\right) = -\frac{32}{243}$

e.g. The mass of the sun is about 2000000000000000000000000000000 kg (2 nonillion kg)
It is much easier to write this as 2×10^{30} kg.

Practice: Work with Exponents

1. What is the base of each power?

a) 5^2 **b)** 2^3
c) $(-3)^4$ **d)** -3^4
e) $\left(\frac{2}{3}\right)^2$ **f)** 2.1^2

2. Write the exponent for each power in question 1.

3. Which expressions are equal to $4 \times 4 \times 4$?

A 3^4 **B** 4^3
C 12 **D** 64

4. Which expression in question 3 is $4 \times 4 \times 4$ written as a power?

5. Which expressions are equal to 2^4?

A 2×4 **B** 4×4
C $2 \times 2 \times 2 \times 2$ **D** 16

6. Which expression in question 5 is 2^4 written in expanded form?

7. Write each expression as a power.

a) $6 \times 6 \times 6 \times 6 \times 6 \times 6 \times 6$
b) 9×9
c) $0.4 \times 0.4 \times 0.4$
d) $(-7) \times (-7) \times (-7) \times (-7) \times (-7)$
e) $(-1.3) \times (-1.3) \times (-1.3) \times (-1.3)$
f) $\left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right) \times \left(\frac{2}{5}\right)$

8. Write each power in expanded form, then evaluate.

a) 3^4 **b)** 5^3
c) $(-2)^2$ **d)** -3^4
e) $\left(\frac{1}{4}\right)^2$ **f)** 0.4^3

9. Evaluate.

a) 6^3 **b)** 2^7
c) -4^2 **d)** $(-2)^6$
e) 1^{12} **f)** $\left(-\frac{4}{5}\right)^2$

10. Use the correct order of operations to evaluate each expression.

a) $2^4 + 3^2$ **b)** $6^3 - 6$
c) $(2 + 5)^2$ **d)** $(2^2 + 5^2)$
e) $6\left(\frac{1}{3}\right)^2$ **f)** $8^2 \div 2^4$

11. Evaluate each expression for the given values of the variables.

a) $3x^4$ $x = 2$
b) $2x^2 + 5$ $x = 3$
c) $4r^2 - r$ $r = 6$
d) $t^2 - 2t$ $t = 4$
e) $m^2 + m - 4$ $m = 3$
f) $x^2 - y^2$ $x = 7, y = 5$

Answers

1. a) 5 **b)** 2 **c)** (–3) **d)** 3 **e)** $\frac{2}{3}$ **f)** 2.1

2. a) 2 **b)** 3 **c)** 4 **d)** 4 **e)** 2 **f)** 2

3. B; D **4.** 4^3 **5.** B; C; D

6. $2 \times 2 \times 2 \times 2$

7. a) 6^7 **b)** 9^2 **c)** 0.4^3 **d)** $(-7)^5$ **e)** $(-1.3)^4$ **f)** $\left(\frac{2}{5}\right)^4$

8. a) $3 \times 3 \times 3 \times 3$; 81 **b)** $5 \times 5 \times 5$; 125
c) $(-2) \times (-2)$; 4 **d)** $-(3 \times 3 \times 3 \times 3)$; –81
e) $\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)$; $\frac{1}{16}$ **f)** $0.4 \times 0.4 \times 0.4$; 0.064

9. a) 216 **b)** 128 **c)** –16 **d)** 64 **e)** 1 **f)** $\frac{16}{25}$

10. a) 25 **b)** 210 **c)** 49 **d)** 29 **e)** $\frac{2}{3}$ **f)** 4

11. a) 48 **b)** 23 **c)** 138 **d)** 8 **e)** 8 **f)** 24

How to Read Powers

e.g. Consider the power 2^3. It can be read in a variety of different ways as shown below.

2^3

- "Two to the exponent three"
- "Two cubed"
- "The third power of two"
- "Two to the third"
- "Two raised to the exponent three"

Note that many people will also say, "Two to the ***power*** three." Technically, this is incorrect because the power is 2^3 ***not*** three. However, it is such a common practice to use the word "power" as if it were synonymous with "exponent" that we have no choice but to accept it.

A Common Mistake that you Should Never Make

NEVER confuse powers with multiplication

e.g. 2^3 ***means*** $2\times2\times2=8$ ***NOT*** $2\times3=6$

If you confuse powers with multiplication, then the mass of the sun would be only 600 kg, which is clearly nonsensical!

Mass of sun = 2×10^{30} kg = 2 times10 multiplied by itself 30 times ***NOT*** $2\times10\times30=600$

Simplifying Expressions involving Powers by writing in Expanded Form

In the following examples, the expressions are simplified (written as a single power) by writing powers in expanded form. This is done to help you remember to ***think about the meaning of powers before you write your answers***.

(a) $3^2\left(3^4\right)=\underbrace{3(3)(3)(3)(3)(3)}_{\text{There are six factors of 3 altogether.}}=3^6$

(b)
$$\begin{aligned}\frac{4^5}{4^3}&=\frac{4(4)(4)(4)(4)}{4(4)(4)}\\&=\left[\frac{4(4)(4)}{4(4)(4)}\right]\left[\frac{4(4)}{1}\right]\\&=[1][4(4)]\\&=4^2\end{aligned}$$

(c)
$$\begin{aligned}\left(5^2\right)^3&=\left(5^2\right)\left(5^2\right)\left(5^2\right)\\&=5(5)(5)(5)(5)(5)\\&=5^6\end{aligned}$$

(d) $x^2\left(x^4\right)=x(x)(x)(x)(x)(x)=x^6$

(e)
$$\begin{aligned}\frac{a^5}{a^3}&=\frac{a(a)(a)(a)(a)}{a(a)(a)}\\&=\left[\frac{a(a)(a)}{a(a)(a)}\right]\left[\frac{a(a)}{1}\right]\\&=[1][a(a)]\\&=a^2\end{aligned}$$

(f)
$$\begin{aligned}\left(s^2\right)^3&=\left(s^2\right)\left(s^2\right)\left(s^2\right)\\&=s(s)(s)(s)(s)(s)\\&=s^6\end{aligned}$$

(g) $3\left(y^3\right)^2$

(h) $\left(3y^3\right)^2$

Understanding the Laws of Exponents

Name of Law	*Law Expressed in Algebraic Form*	*Law Expressed in Verbal Form*	*Example Showing why Law Works*
Product Rule	$a^x a^y = a^{x+y}$	To ***multiply*** two powers with the ***same base, keep the base*** and ***add the exponents***.	$a^2a^4 = (a)(a)(a)(a)(a)(a) = a^6$ ***Two*** factors of *a* multiplied by ***four*** factors of *a* gives ***six*** factors of *a*.
Quotient Rule	$\dfrac{a^x}{a^y} = a^{x-y}$	To ***divide*** two powers with the ***same base, keep the base*** and ***subtract the exponents***.	$\dfrac{a^5}{a^2} = \dfrac{(a)(a)(a)(a)(a)}{(a)(a)} = a^3$ ***Five*** factors of *a* divided by ***two*** factors of *a* leaves ***three*** factors of *a*. (Two factors of *a* in the numerator divide out with two factors of *a* in the denominator.)
Power of a Power Rule	$(a^x)^y = a^{xy}$	To ***raise*** a power to an exponent, ***keep the base*** and ***multiply*** the exponents.	$(a^3)^4 = (a^3)(a^3)(a^3)(a^3) = a^{12}$

Examples

Use the laws of exponents to simplify each of the following expressions.

(a) $x^2(x^4) = x^{2+4} = x^6$

(b) $\dfrac{y^5}{y^3} = y^{5-3} = y^2$

(c) $x^2(y^4)$ cannot be simplified because the bases are different

(d) $(a^3)^6 = a^{3\times6} = a^{18}$

(e) $\dfrac{x^5}{y^3}$ cannot be simplified because the bases are different

(f) $5(x^2)^3 = 5(x^{2\times3}) = 5x^6$

(g) $(5x^2)^3 = (5x^2)(5x^2)(5x^2)$
$= 5(5)(5)(x^2)(x^2)(x^2)$
$= 125x^{2+2+2}$
$= 125x^6$

Another Law of Exponents

Example (g) above suggests the following shortcut: $(5x^2)^3 = 5^3(x^2)^3 = 125x^{2\times3} = 125x^6$. In general, this can be expressed as follows:

Power of a Product

$$(ab)^x = a^x b^x$$

To ***raise a product to an exponent, raise each factor*** in the product ***to the exponent***.

e.g. $(m^3n^4)^6 = (m^3)^6(n^4)^6 = m^{18}n^{24}$

A Big Example

Use the laws of exponents to simplify $\dfrac{2ab^2(3a^3b^3)}{(4ab^2)^4}$

$$\frac{2ab^2(3a^3b^7)}{(4ab^2)^4} = \frac{2(3)a^1a^3b^2b^7}{4^4a^4(b^2)^4} = \frac{6a^{1+3}b^{2+7}}{256a^4b^{2\times4}} = \frac{6a^4b^9}{256a^4b^8} = \left(\frac{6}{256}\right)\left(\frac{a^4}{a^4}\right)\left(\frac{b^9}{b^8}\right) = \left(\frac{3}{128}\right)(1)b^{9-8} = \frac{3}{128}b$$

Since multiplication can be performed in any order, the ***like factors*** can be "put together."

Note that multiplication is both ***commutative*** and ***associative***.

$\dfrac{3}{128}b = \dfrac{3b}{128}$ because

$\dfrac{3}{128}b = \dfrac{3}{128}\left(\dfrac{b}{1}\right) = \dfrac{3b}{128}$

Practice: Discover the Exponent Laws

1. Write each expression in expanded form. Then write as a single power.

a) $7^2 \times 7^4$ **b)** $3^5 \times 3^3$
c) 5×5^2 **d)** $3^2 \times 3^4 \times 3^3$
e) $(-2)^2 \times (-2)^3$ **f)** $(-1)^3 \times (-1)^2 \times (-1)$
g) $0.5^3 \times 0.5^2$ **h)** $\left(\frac{1}{2}\right) \times \left(\frac{1}{2}\right)^3$

2. Evaluate each expression in question 1.

3. Write each expression in expanded form. Then write as a single power.

a) $8^6 \div 8^4$ **b)** $5^5 \div 5^3$
c) $7^7 \div 7^2$ **d)** $4^8 \div 4^5 \div 4$
e) $(-9)^7 \div (-9)^6$ **f)** $0.1^6 \div 0.1^4$
g) $(-0.3)^4 \div (-0.3)$ **h)** $\left(\frac{2}{3}\right)^5 \div \left(\frac{2}{3}\right)^3$

4. Evaluate each expression in question 3.

5. Write each expression in expanded form. Then, write as a single power.

a) $(2^2)^4$ **b)** $(6^2)^2$
c) $(3^3)^2$ **d)** $[(-2)^4]^3$
e) $[(-1)^8]^6$ **f)** $[(-1)^5]^7$
g) $(0.3^2)^2$ **h)** $\left[\left(\frac{2}{5}\right)^2\right]^2$

6. Evaluate each expression in question 5.

7. Use the exponent laws to simplify each expression. Then, evaluate.

a) $4^3 \times 4^4 \div 4^5$ **b)** $8^7 \div 8^7 \times 8$
c) $\dfrac{9^6 \times 9^3}{9^7}$ **d)** $\dfrac{6^5 \times 6^2}{6 \times 6^3}$
e) $(2^4)^2 \times 2^3$ **f)** $\dfrac{(3^2)^4 \times 3^3}{3^8}$
g) $0.2^6 \times 0.2^5 \div (0.2^2)^5$ **h)** $[(-4)^3]^4 \div [(-4)^2]^5$

8. Simplify.

a) $b^5 \times b^3$ **b)** $p^4 \times p$
c) $w^5 \div w^2$ **d)** $x^8 \div x^4$
e) $(m^5)^2$ **f)** $(k^2)^3 \times k^2$
g) $g^5 \times g^5 \div g^7$ **h)** $(a^6)^3 \div (a^5)^2$

9. Simplify.

a) $a^4b^5 \times ab^3$ **b)** $m^2n^4 \times m^3n^3$
c) $p^6q^5 \div (p^3q^2)$ **d)** $6xy^2 \div (2y)$
e) $(gh^4)^3$ **f)** $2k^2m^3 \times (2k^2)^2$
g) $\dfrac{(2g^5h^3)^2}{2gh^6}$ **h)** $\dfrac{6b^2d \times 3b^2d^2}{(3bd)^2}$

10. If $\left(\frac{1}{2}\right)^x = 0.015625$, determine the value of x.

Answers

1. a) $7 \times 7 \times 7 \times 7 \times 7 \times 7$; 7^6 **b)** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$; 3^8
c) $5 \times 5 \times 5$; 5^3 **d)** $3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3 \times 3$; 3^9
e) $(-2) \times (-2) \times (-2) \times (-2) \times (-2)$; $(-2)^5$
f) $(-1) \times (-1) \times (-1) \times (-1) \times (-1) \times (-1)$; $(-1)^6$
g) $0.5 \times 0.5 \times 0.5 \times 0.5 \times 0.5$; 0.5^5
h) $\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)$; $\left(\frac{1}{2}\right)^4$

2. a) 117 649 **b)** 6561 **c)** 125 **d)** 19 683
e) –32 **f)** 1 **g)** 0.031 25 **h)** $\frac{1}{16}$

3. a) $\dfrac{8\times8\times8\times8\times8\times8}{8\times8\times8\times8}$; 8^2 **b)** $\dfrac{5\times5\times5\times5\times5}{5\times5\times5}$; 5^2
c) $\dfrac{7\times7\times7\times7\times7\times7\times7}{7\times7}$; 7^5 **d)** $\dfrac{4\times4\times4\times4\times4\times4\times4\times4}{(4\times4\times4\times4\times4)\times4}$; 4^2
e) $\dfrac{(-9)\times(-9)\times(-9)\times(-9)\times(-9)\times(-9)\times(-9)}{(-9)\times(-9)\times(-9)\times(-9)\times(-9)\times(-9)}$; $(-9)^1$
f) $\dfrac{0.1\times0.1\times0.1\times0.1\times0.1\times0.1}{0.1\times0.1\times0.1\times0.1}$; 0.1^2
g) $\dfrac{(-0.3)\times(-0.3)\times(-0.3)\times(-0.3)}{(-0.3)}$; $(-0.3)^3$
h) $\dfrac{\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)}{\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)\times\left(\frac{2}{3}\right)}$; $\left(\frac{2}{3}\right)^2$

4. a) 64 **b)** 25 **c)** 16 807 **d)** 16
e) –9 **f)** 0.01 **g)** –0.027 **h)** $\frac{4}{9}$

5. a) $(2^2) \times (2^2) \times (2^2) \times (2^2)$; 2^8
b) $(6^2) \times (6^2)$; 6^4
c) $(3^3) \times (3^3)$; 3^6
d) $(-2)^4 \times (-2)^4 \times (-2)^4$; $(-2)^{12}$
e) $(-1)^8 \times (-1)^8 \times (-1)^8 \times (-1)^8 \times (-1)^8 \times (-1)^8$; $(-1)^{48}$
f) $(-1)^5\times(-1)^5\times(-1)^5\times(-1)^5\times(-1)^5\times(-1)^5\times(-1)^5$; $(-1)^{35}$
g) $(0.3^2) \times (0.3^2)$; 0.3^4
h) $\left(\frac{2}{5}\right)^2 \times \left(\frac{2}{5}\right)^2$; $\left(\frac{2}{5}\right)^4$

6. a) 256 **b)** 1296 **c)** 729 **d)** 4096
e) 1 **f)** –1 **g)** 0.0081 **h)** $\frac{16}{625}$

7. a) 4^2; 16 **b)** 8^1; 8 **c)** 9^2; 81
d) 6^3; 216 **e)** 2^{11}; 2048 **f)** 3^3; 27
g) 0.2^1; 0.2 **h)** $(-4)^2$; 16

8. a) b^8 **b)** p^5 **c)** w^3 **d)** x^4
e) m^{10} **f)** k^8 **g)** g^3 **h)** a^8

9. a) a^5b^8 **b)** m^5n^7 **c)** p^3q^3 **d)** $3xy$
e) g^3h^{12} **f)** $8k^6m^3$ **g)** $2g^9$ **h)** $2b^2d$

10. Use trial and error.

Pre-AP Enrichment: Powers with Zero and Negative Exponents

How the Heck can Exponents be Zero or Negative?

Consider the following pattern:

Power	**Value**	**Meaning**
10^5	100000 ↓ ÷10	$10(10)(10)(10)(10)$
10^4	10000 ↓ ÷10	$10(10)(10)(10)$
10^3	1000 ↓ ÷10	$10(10)(10)$
10^2	100 ↓ ÷10	$10(10)$
10^1	10 ↓ ÷10	10
10^0	1 ↓ ÷10	1
10^{-1}	0.1 ↓ ÷10	$\frac{1}{10}$
10^{-2}	0.01 ↓ ÷10	$\frac{1}{10(10)} = \frac{1}{10^2}$
10^{-3}	0.001 ↓ ÷10	$\frac{1}{10(10)(10)} = \frac{1}{10^3}$
10^{-4}	0.0001 ↓ ÷10	$\frac{1}{10(10)(10)(10)} = \frac{1}{10^4}$
10^{-5}	0.00001	$\frac{1}{10(10)(10)(10)(10)} = \frac{1}{10^5}$

1. What happens to the value of the power when the exponent is decreased by 1? Describe this both in terms of mathematical operations as well as how the "appearance" of the value changes.

2. What happens to the number of factors of 10 each time the exponent is decreased by 1?

3. Keeping in mind your answers to questions 1 and 2, describe the effect of reducing the exponent of a power by one.

4. Use the example at the left to speculate on the following:

$x^0 =$ ______ for all values of x.

$x^{-n} =$ ______ for all values of x and n.

Using Laws of Exponents to Understand Zero and Negative Exponents

1. Why is $x^0 = 1$?

A number divided by itself is 1!

$$x^0 = x^{n-n} = \frac{x^n}{x^n} = 1$$

Using the quotient rule.

$$\boxed{\therefore x^0 = 1}$$

2. Why is $x^{-n} = \frac{1}{x^n}$?

Using the result from 1, i.e. $x^0 = 1$.

$$x^{-n} = x^{0-n} = \frac{x^0}{x^n} = \frac{1}{x^n}$$

Using the quotient rule.

$$\boxed{\therefore x^{-n} = \frac{1}{x^n}}$$

Most students are dumbstruck when they are taught that $x^0 = 1$ and $x^{-n} = \frac{1}{x^n}$. As counterintuitive as these results might seem, we have to accept them if we would like to extend our laws of exponents to more general situations.

Exercises

True or False? Explain your answer.

1. $10^{-2} = \dfrac{1}{100}$

2. $\left(-\dfrac{1}{5}\right)^{-1} = 5$

3. $3^{-2} \cdot 2^{-1} = 6^{-3}$

4. $\dfrac{3^{-2}}{3^{-1}} = \dfrac{1}{3}$

5. $23.7 = 2.37 \times 10^{-1}$

6. $0.000036 = 3.6 \times 10^{-5}$

Evaluate.

7. 3^{-1} **8.** 3^{-3} **9.** $(-2)^{-4}$

10. $(-3)^{-4}$ **11.** -4^{-2} **12.** -2^{-4}

13. $\dfrac{5^{-2}}{10^{-2}}$ **14.** $\dfrac{3^{-4}}{6^{-2}}$

15. $\left(\dfrac{5}{2}\right)^{-3}$ **16.** $\left(\dfrac{4}{3}\right)^{-2}$ **17.** $6^{-1} + 6^{-1}$

18. $2^{-1} + 4^{-1}$ **19.** $\dfrac{10}{5^{-3}}$ **20.** $\dfrac{1}{25 \cdot 10^{-4}}$

21. $\dfrac{1}{4^{-3}} + \dfrac{3^2}{2^{-1}}$ **22.** $\dfrac{2^3}{10^{-2}} - \dfrac{2}{7^{-2}}$

Simplify. Write each answer WITHOUT using negative exponents.

23. $x^{-1}x^2$ **24.** $y^{-3}y^5$

25. $-2x^2 \cdot 8x^{-6}$ **26.** $5y^5(-6y^{-7})$

27. $-3a^{-2}(-2a^{-3})$ **28.** $(-b^{-3})(-b^{-5})$

29. $\dfrac{u^{-5}}{u^3}$ **30.** $\dfrac{w^{-4}}{w^6}$

31. $\dfrac{8t^{-3}}{-2t^{-5}}$ **32.** $\dfrac{-22w^{-4}}{-11w^{-3}}$

33. $\dfrac{-6x^5}{-3x^{-6}}$ **34.** $\dfrac{-51y^6}{17y^{-9}}$

35. $(x^2)^{-5}$ **36.** $(y^{-2})^4$ **37.** $(a^{-3})^{-3}$

38. $(b^{-5})^{-2}$ **39.** $(2x^{-3})^{-4}$ **40.** $(3y^{-1})^{-2}$

41. $(4x^2y^{-3})^{-2}$ **42.** $(6s^{-2}t^4)^{-1}$

43. $\left(\dfrac{2x^{-1}}{y^{-3}}\right)^{-2}$ **44.** $\left(\dfrac{a^{-2}}{3b^3}\right)^{-3}$

45. $\left(\dfrac{2a^{-3}}{ac^{-2}}\right)^{-4}$ **46.** $\left(\dfrac{3w^2}{w^4x^3}\right)^{-2}$

51. $(x^{-2})^{-3} + 3x^7(-5x^{-1})$

52. $(ab^{-1})^2 - ab(-ab^{-3})$

53. $\dfrac{a^3b^{-2}}{a^{-1}} + \left(\dfrac{b^6a^{-2}}{b^5}\right)^{-2}$

54. $\left(\dfrac{x^{-3}y^{-1}}{2x}\right)^{-3} + \dfrac{6x^9y^3}{-3x^{-3}}$

Use the "Power of a Quotient Rule" for 43, 44, 45, 46, 53 and 54

$$\left(\frac{a}{b}\right)^c = \frac{a^c}{b^c}$$

Answers

1. True **2.** False, -5 **3.** False, $\frac{1}{18}$ **4.** True **5.** False, 2.37×10^1 **6.** True **7.** $\frac{1}{3}$ **8.** $\frac{1}{27}$

9. $\frac{1}{16}$ **10.** $\frac{1}{81}$ **11.** $-\frac{1}{16}$ **12.** $-\frac{1}{16}$ **13.** 4 **14.** $\frac{36}{81} = \frac{4}{9}$ **15.** $\frac{8}{125}$ **16.** $\frac{9}{16}$

17. $\frac{2}{6} = \frac{1}{3}$ **18.** $\frac{3}{4}$ **19.** 1250 **20.** 400 **21.** 82 **22.** 702 **23.** x **24.** y^2

25. $\frac{-16}{x^4}$ **26.** $\frac{-30}{y^2}$ **27.** $\frac{6}{a^5}$ **28.** $\frac{1}{b^8}$ **29.** $\frac{1}{u^8}$ **30.** $\frac{1}{w^{10}}$ **31.** $-4t^2$ **32.** $\frac{2}{w}$

33. $2x^{11}$ **34.** $-3y^{15}$ **35.** $\frac{1}{x^{10}}$ **36.** $\frac{1}{y^8}$ **37.** a^9 **38.** b^{10} **39.** $\frac{x^{12}}{16}$ **40.** $\frac{y^2}{9}$

41. $\frac{y^6}{16x^4}$ **42.** $\frac{s^2}{6t^4}$ **43.** $\frac{x^2}{4y^6}$ **44.** $27a^6b^9$ **45.** $\frac{a^{16}}{16c^8}$ **46.** $\frac{w^4x^6}{9}$ **51.** $-14x^6$ **52.** $\frac{2a^2}{b^2}$

53. $\frac{2a^4}{b^2}$ **54.** $6x^{12}y^3$

Putting all the Operations Together: The Distributive Property

Review – Operating with Integers

Adding and Subtracting Integers

- Addition and subtraction of integers always involve ***MOVEMENTS***

  Add a Positive Value or Subtract a Negative Value
  +(+) ***or*** –(–) → ***GAIN*** (move ***up*** or ***right***)
  Add a Negative Value or Subtract a Positive Value
  +(–) ***or*** –(+) → ***LOSS*** (move ***down*** or ***left***)
- Movements on a number line
- Moving from one floor to another using an elevator
- Loss/gain of yards in football
- Loss/gain of money in bank account or stock market
- It is ***NOT POSSIBLE*** to predict the sign of the answer to an addition or subtraction only by counting the number of negative signs!

$2+(-5)$	$5+(-2)$	$8-(+5)$	$-2-(-5)$
$=2-5$	$=5-2$	$=8-5$	$=-2+5$
$=-3$	$=3$	$=3$	$=3$

Multiplying and Dividing Integers

- Multiplication is ***repeated addition***

  e.g. 5(–2) = 5 groups of –2
  = (–2)+(–2)+(–2)+(–2)+(–2) = –10
- Division is the ***opposite of*** multiplication

  e.g. –10÷(–2) = How many groups of –2 in –10?
  = 5

  Multiply or Divide Two Numbers of Like Sign
  (+)(+) ***or*** (–)(–) → ***POSITIVE RESULT***

  Multiply or Divide Two Numbers of Unlike Sign
  (+)(–) ***or*** (–)(+) → ***NEGATIVE RESULT***
- The sign of the answer to a multiplication or division is ***DETERMINED ENTIRELY*** by the signs of the numbers!

$2(-5)$	$-12\div 6$	$-12\div(-6)$
$=-10$	$=-2$	$=2$

Understanding how to Multiply a Monomial by a Binomial

Consider the expression shown below. Although BEDMAS would tell us to perform the operations within the brackets first, we ***cannot*** do so because 4*x* and 2 are unlike terms. Nevertheless, we can still deal with this expression in a variety of ways.

$$3(\overbrace{4x+2}^{\text{Cannot be simplified because } 4x \text{ and } 2 \text{ are unlike terms.}})$$

Multiply 3 by $(4x+2)$ using the Definition of Multiplication

Recall that multiplication is a short form for repeated addition:

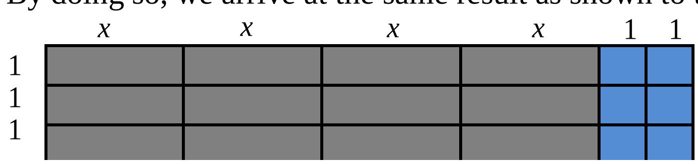
e.g. $3a=a+a+a$

Therefore,

$$\begin{aligned}&3(4x+2)\\&=(4x+2)+(4x+2)+(4x+2)\\&=4x+2+4x+2+4x+2\\&=4x+4x+4x+2+2+2\\&=12x+6\end{aligned}$$

Multiply 3 by $(4x+2)$ using an Area Model (Algebra Tiles)

We can calculate the area of the following figure in two different ways. By doing so, we arrive at the same result as shown to the left.



Area = width × length $=3(4x+2)=12x+6$

Area = $x+x+x+x+x+x+x+x+x+x+x+x+1+1+1+1+1+1$
$= 12x + 6$

Therefore, $3(4x+2)$.

A Shortcut for Multiplying a Monomial by a Polynomial

We performed the multiplication $3(4x+2)$ using two different methods and in both cases, we found that the product was $12x+6$. Unfortunately, while both methods allowed us to "see" clearly what the product should be, a great deal of time was required to arrive at the answer. To resolve this problem, we can use the following:

The Distributive Property

This property is called the "distributive property" because the monomial *a* is ***distributed*** to each term enclosed in parentheses.

Factored Form → $a(x+y)=ax+ay$ ← Expanded Form

To ***expand*** the product of a monomial by a binomial, ***multiply each term*** of the binomial by the monomial. In other words, ***multiply each term in the brackets by a***.

Examples

1. Expand each of the following. Then simplify if possible.

(a)
$$-5(3x+y)=-5(3x)+(-5y)$$
$$=-15x-5y$$

(b)
$$-7(2a-5b)=-7(2a)-(-7)(5b)$$
$$=-14a-(-35b)$$
$$=-14a+35b$$

(c)
$$x(-3x^2-y^2)=x(-3x^2)-x(y^2)$$
$$=-3x^3-xy^2$$

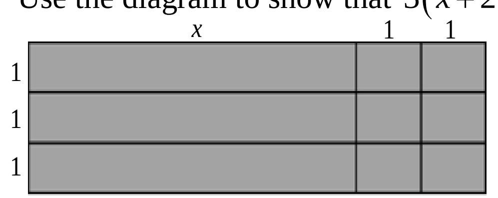
(d)
$$-3(3x+2y)-5(-4x-6y+1)$$
$$=-9x-6y-(-20x-30y+5)=-9x-6y+(20x+30y-5)$$
$$=-9x-6y+20x+30y-5=11x+24y-5$$

Shortcut that must be used with Caution

$(-\times+)=-$ $\qquad$ $(-\times-)=+$

$$-7(2a-5b)=-14a+35b$$

2. Use the diagram to show that $3(x+2)=3x+6$.



Solution

Length = $x + 2$, $\quad$ Width = 3 $\qquad$ Area = $3(x+2)$

But the area can also be calculated as follows: Area = $x+x+x+1+1+1+1+1+1=3x+6$

Therefore, $3(x+2)=3x+6$

3. Use the ***distributive property*** to evaluate the following products ***without*** using a calculator.

(a) $98(101)$
$$98(101)=98(100+1)$$
$$=98(100)+98(1)$$
$$=9800+98$$
$$=9898$$

(b) $101(1101)$
$$101(1101)=101(1000+100+1)$$
$$=101(1000)+101(100)+101(1)$$
$$=101000+10100+101$$
$$=111100+101$$
$$=111201$$

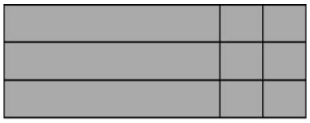
(c) $98(99)$
$$98(99)=98(100-1)$$
$$=98(100)-98(1)$$
$$=9800-98$$
$$=9702$$

Practice: The Distributive Property

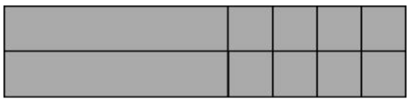
1. Copy and complete the table for each rectangle.

Rectangle	Width	Length	Area	Equation

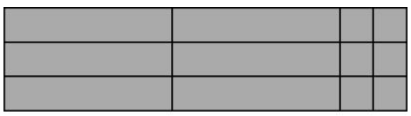
A



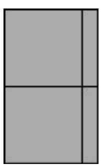
B



C



D



2. Model each expression with algebra tiles. Then, expand each expression.

a) $5(x + 2)$ **b)** $4(2x + 3)$
c) $x(x + 4)$ **d)** $2x(2x + 5)$

3. Which expression is equal to $6(x - 4)$?

A $6x - 4$ **B** $6x + 4$
C $x - 24$ **D** $6x - 24$

4. Use the distributive property to expand.

a) $3(g + 4)$ **b)** $2(a + 5)$
c) $6(x - 3)$ **d)** $5(b - 1)$
e) $4(3 - r)$ **f)** $-7(q + 3)$
g) $-2(6 - t)$ **h)** $-4(-w - 5)$

5. Expand.

a) $b(b + 1)$ **b)** $m(m + 4)$
c) $x(x - 2)$ **d)** $a(a + 1)$
e) $r(3r + 5)$ **f)** $q(2q + 3)$
g) $k(6 - k)$ **h)** $w(4w - 5)$

6. Expand.

a) $3p(p + 4)$ **b)** $2s(s + 2)$
c) $4x(2x - 1)$ **d)** $6b(3b + 1)$
e) $-r(-5r + 2)$ **f)** $-y(2y - 7)$
g) $5c(8 - 2c)$ **h)** $-3w(2w - 1)$

7. Expand.

a) $(d + 3) \times 2$ **b)** $(k + 1) \times 4$
c) $(w - 2) \times 5$ **d)** $(u - 1) \times (-3)$
e) $(2q + 5) \times 6$ **f)** $(-p + 4) \times (-2)$
g) $(5 - z)(3z)$ **h)** $(6w - 4)(-3w)$

8. Expand.

a) $3(x^2 + x - 4)$
b) $2(m^2 - 3m + 5)$
c) $-4(b^2 - 2b - 3)$
d) $5c(c^2 - 6c - 1)$
e) $-3h(4 - h^2)$
f) $(n^2 + 4n + 3)(-2)$
g) $(5t^2 - 2t)(-t)$
h) $(w^2 + 2w - 5)(4w)$

9. Expand and simplify.

a) $2(b + 3) + 5(b + 4)$
b) $3(p - 2) + 6(p + 1)$
c) $-5(m + 5) + 2(m - 7)$
d) $-(d - 4) - 4(d + 2)$

10. Expand and simplify.

a) $4[b + 3(b + 1)]$
b) $2[3(a + 4) - 4]$
c) $5[4s - (s + 2)]$
d) $3[-2(6 - t) + 5t]$

11. Use the distributive property to evaluate the following without using a calculator.

a) 90(109)
b) 91(120)
c) 900(99999)
d) 99(1011)

12. Write an expression for the total amount of money in two different ways. Then evaluate using the distributive property.

13. Expand the product $(x-3)(x+7)$.

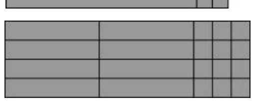
Answers

1.

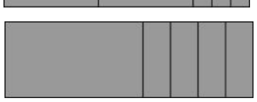
Rectangle	Width	Length	Area	Equation
A	3	$x + 2$	$3x + 6$	$3(x + 2) = 3x + 6$
B	2	$x + 4$	$2x + 8$	$2(x + 4) = 2x + 8$
C	3	$2x + 2$	$6x + 6$	$3(2x + 2) = 6x + 6$
D	$2x$	$x + 1$	$2x^2 + 2x$	$2x(x + 1) = 2x^2 + 2x$

2. a) $5x + 10$

b) $8x + 12$



c) $x^2 + 4x$



d) $4x^2 + 10x$

3. D: $6x - 24$

4. a) $3g + 12$ **b)** $2a + 10$ **c)** $6x - 18$ **d)** $5b - 5$
e) $12 - 4r$ **f)** $-7q - 21$ **g)** $-12 + 2t$ **h)** $4w + 20$

5. a) $b^2 + b$ **b)** $m^2 + 4m$ **c)** $x^2 - 2x$ **d)** $a^2 + a$
e) $3r^2 + 5r$ **f)** $2q^2 + 3q$ **g)** $6k - k^2$ **h)** $4w^2 - 5w$

6. a) $3p^2 + 12p$ **b)** $2s^2 + 4s$ **c)** $8x^2 - 4x$ **d)** $18b^2 + 6b$
e) $5r^2 - 2r$ **f)** $-2y^2 + 7y$ **g)** $40c - 10c^2$ **h)** $-6w^2 + 3w$

7. a) $2d + 6$ **b)** $4k + 4$ **c)** $5w - 10$ **d)** $-3u + 3$
e) $12q + 30$ **f)** $2p - 8$ **g)** $15z - 3z^2$ **h)** $-18w^2 + 12w$

8. a) $3x^2 + 3x - 12$ **b)** $2m^2 - 6m + 10$
c) $-4b^2 + 8b + 12$ **d)** $5c^3 - 30c^2 - 5c$
e) $-12h + 3h^3$ **f)** $-2n^2 - 8n - 6$
g) $-5t^3 + 2t^2$ **h)** $4w^3 + 8w^2 - 20w$

9. a) $7b + 26$ **b)** $9p$ **c)** $-3m - 39$ **d)** $-5d - 4$

10. a) $16b + 12$ **b)** $6a + 16$ **c)** $15s - 10$ **d)** $-36 + 21t$

11. a) 90(100+9) **b)** 91(100+20) **c)** 900(100000−1) **d)** 99(1000+10+1)

12. $3(0.10) + 3(0.25) + 3(1) = 3(0.10 + 0.25 + 1)$ **13. Hint:** Distribute $x-3$ to x and 7

Summary of Simplifying Algebraic Expressions

Review of Simplifying Algebraic Expressions

Adding and Subtracting Polynomials with no Brackets

1. Collect like terms. ***Remember that the operation must "move" with the term!***
2. Use the rules for adding and subtracting integers. (GAINS/LOSSES)

Examples

$$\begin{aligned}&-5a-6b+3a-4b\\=&-5a+3a-6b-4b\\=&-2a-10b\end{aligned}$$

$$\begin{aligned}&-2x^2y+3xy-7x^2y-8xy\\=&-2x^2y-7x^2y+3xy-8xy\\=&-9x^2y-5xy\end{aligned}$$

Adding and Subtracting Polynomials with Brackets

1. If a bracket is preceded by a "+" sign or no sign, the brackets can simply be removed because addition is ***insensitive*** to order.
2. If a bracket is preceded by a "–" sign, brackets cannot be removed because subtraction is sensitive to order. After ***adding the opposite***, the brackets can be removed. This works because − = + (−).
3. Collect like terms.
4. Use the rules for adding and subtracting integers. (GAINS/LOSSES)

Example

$$\begin{aligned}&(-5a+6b)-(3a-4b)\\=&-5a+6b+(-3a+4b)\\=&-5a+6b+(-3a)+4b\\=&-5a+6b-3a+4b\\=&-5a-3a+6b+4b\\=&-8a+10b\end{aligned}$$

Add the opposite of $3a-4b$

Brackets can be removed now because the operation is "+."

The Distributive Property

1. Look for an expression in brackets containing two or more terms (usually the terms are unlike) and a factor outside the brackets.
2. Multiply each term in the brackets by the ***factor*** outside the brackets.

Examples

$$\begin{aligned}x(-3x^2-y^2)&=x(-3x^2)-x(y^2)\\&=-3x^3-xy^2\end{aligned}$$

$$\begin{aligned}&(-5a+6b)-(3a-4b)\\=&-5a+6b-1(3a-4b)\\=&-5a+6b-3a+4b\\=&-5a-3a+6b+4b\\=&-8a+10b\end{aligned}$$

The distributive property can be used as an ***alternative*** to adding the opposite. This is based on the property $1x=x$ (i.e. multiplying by 1 has no effect).

Multiplying and Dividing Monomials

1. Make sure there is ***only one term***. If there are two or more terms, make sure that you work on each term separately.
2. Put ***like factors*** together. You are allowed to do this because multiplication can be performed in any order.
3. Use the laws of exponents.

Example

$$\frac{2ab^2(3a^3b^7)}{(4ab^2)^4}=\frac{2(3)a^1a^3b^2b^7}{4^4a^4(b^2)^4}=\frac{6a^{1+3}b^{2+7}}{256a^4b^{2\times4}}=\frac{6a^4b^9}{256a^4b^8}$$

$$=\left(\frac{6}{256}\right)\left(\frac{a^4}{a^4}\right)\left(\frac{b^9}{b^8}\right)=\left(\frac{3}{128}\right)(1)b^{9-8}=\frac{3}{128}b$$

A more Complicated Example Involving the Distributive Property

$$\begin{aligned}&-3a^2b(-6abc+9a^3b^4-7bc)\\=&3a^2b(6abc)-3a^2b(9a^3b^4)+3a^2b(7bc)\\=&18a^3b^2c-27a^5b^5+21a^2b^2c\end{aligned}$$

Understanding the Distributive Property from a Different Point of View

Consider the box of chocolates shown at the right. As shown in the table given below, a variable is used to represent the type of chocolate.

Variable	*What it Represents*	*Total Number of this type in a Box*
x	One 	$8x$
y	One 	y
z	One 	$2z$
u	One 	$2u$
v	One 	$2v$
w	One 	$5w$

1. Write an algebraic expression that expresses the total number of chocolates in one box.

$$8x + y + 2z + 2u + 2v + 5w$$

2. There are 1000 boxes of these chocolates in a warehouse. Write an algebraic expression that represents the total number of chocolates in the warehouse.

$$1000(8x + y + 2z + 2u + 2v + 5w)$$

3. Now use the distributive property to expand your expression. Does the answer make sense?

$$1000(8x + y + 2z + 2u + 2v + 5w)$$
$$= 8000x + 1000y + 2000z + 2000u + 2000v + 5000w$$

The answer makes sense because in 1000 boxes of chocolates, there should be 8000 of type x, 1000 of type y, 2000 of type z, 2000 of type u, 2000 of type v and 5000 of type w.

The Product of Two Binomials

The distributive property can be exploited to expand the product of any number of polynomials. To understand how this is done, it is sufficient to examine the simple case of multiplying two binomials. This simple case illustrates the essential ideas that can be applied to any product of polynomials.

Example

$$\underbrace{(-3x-5y)}(3x^2-y^2)$$

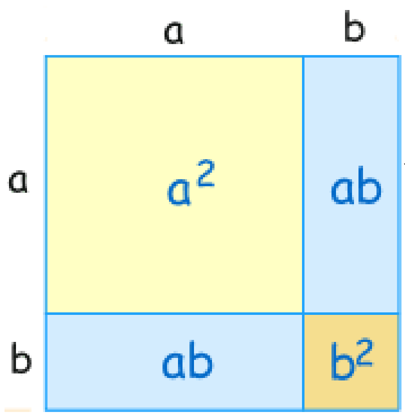
$$=3x^2(-3x-5y)-y^2(-3x-5y)$$

$$=-9x^3-15x^2y+3xy^2+5y^3$$

Although the expression $-3x-5y$ may look a little complicated, it represents the ***result*** of performing a calculation. In other words, once all the operations are performed, the result is a ***single value***. Therefore, the distributive property allows us to distribute this value to each of the terms contained within the second set of parentheses.

A Geometric View

The product $(a+b)(a+b)$ can be represented geometrically as the area of a square with side lengths $a+b$.



Let A represent the area of a square with side lengths $a+b$.

Then $A=lw=(a+b)(a+b)$.

However, the area is also equal to the sum of the areas of the smaller shapes shown.

Therefore, $A=a^2+ab+ab+b^2=a^2+2ab+b^2$

Since $A=(a+b)(a+b)$ **AND** $A=a^2+2ab+b^2$, it follows that

$(a+b)(a+b)=a^2+2ab+b^2$.

This can also be written as $(a+b)^2=a^2+2ab+b^2$.

FOIL: A Mnemonic Shortcut for Multiplying two Binomials

For the sake of performing speedy calculations, many clever shortcuts have been created throughout the years. The following shortcut, which is called "FOIL," provides us with a quick way of expanding the product of two binomials. ***This shortcut is very helpful as long as it is not used as a substitute for thinking!*** Even when the shortcut is being used, it is still important to understand the underlying mathematical ideas that justify the algorithms that we use.

$$(-3x-5y)(3x^2-y^2)$$

FOIL
F → "First"
O → "Outside"
I → "Inside"
L → "Last"

1. Multiply the "first" terms
2. Multiply the "outside" terms
3. Multiply the "inside" terms
4. Multiply the "last" terms

$$=-3x(3x^2)+(-3x)(-y^2)+(-5y)(3x^2)+(-5y)(-y^2)$$

$$=-9x^3+3xy^2-15x^2y+5y^3$$

How to Expand the Product of any two Polynomials – FOIL is FOILED!

If your thinking is confined to the world of mnemonics, you will eventually find mathematics impossibly difficult. The "FOIL" mnemonic, for example, is very useful for expanding the product of two binomials but it fails miserably in all other situations. To understand the general case of multiplying any two polynomials, we can appeal once again to the distributive property. To be sure, fundamental principles such as the distributive property are far more useful in understanding mathematics than even the cleverest mnemonics!

Examples

$$(x+y)(x^2-xy+y^2)$$
$$=x^2(x+y)-xy(x+y)+y^2(x+y)$$
$$=x^3+x^2y-x^2y-xy^2+xy^2+y^3$$
$$=x^3+y^3$$

This shows how the distributive property can be applied to expand a product of polynomials. Notice that there are SIX terms in the expansion (before simplification).

The above example suggests that the product can be obtained by finding all possible combinations of products.
This is shown in the next example:

$$(x+y)(x^2-xy+y^2)$$
$$=x^3-x^2y+xy^2+x^2y-xy^2+y^3$$
$$=x^3-x^2y+x^2y+xy^2-xy^2+y^3$$
$$=x^3+y^3$$

The same answer is obtained simply by forming all possible products. Notice that the total number of products is $2\times3=6$.

Exercises

See the next page.

A

1. Find the product.

a) $(x+1)(x+5)$ **b)** $(x+4)(x+3)$ **c)** $(a+4)(a+4)$
d) $(y+5)(y+6)$ **e)** $(x-4)(x-3)$ **f)** $(a-4)(a-2)$
g) $(b-1)(b-5)$ **h)** $(y-9)(y-9)$ **i)** $(x-6)(x+3)$
j) $(c+2)(c-8)$ **k)** $(t+10)(t-10)$ **l)** $(q-2)(q+5)$

2. Expand. Verify the solution by substituting 1 for the variable.

a) $(c+3)(c-4)$ **b)** $(x+2)(x-5)$ **c)** $(y+6)(y-2)$
d) $(a+9)(a-5)$ **e)** $(x-3)(x+3)$ **f)** $(b-7)(b+10)$
g) $(y-12)(y+3)$ **h)** $(x-7)(x+1)$ **i)** $(4+x)(7-x)$
j) $(2-y)(3-y)$ **k)** $(x+7)(x+7)$ **l)** $(b-8)(b+8)$

3. Expand and simplify.

a) $(x+5)(2x+1)$ **b)** $(3y+1)(y+2)$ **c)** $(x-1)(2x-1)$
d) $(a-3)(2a-5)$ **e)** $(5y-7)(y+3)$ **f)** $(x-5)(4x+3)$
g) $(3x-4)(3x-4)$ **h)** $(1-6t)(4+5t)$ **i)** $(3a-5)(3a+5)$

4. Expand the following.

a) $2(x+3)(x+5)$ **b)** $4(x-9)(x+5)$
c) $-(a+3)(a-2)$ **d)** $10(x+7)(x-5)$
e) $3(2x-1)(3x-2)$ **f)** $-2(4y+1)(y-3)$
g) $0.5(x-1)(x+3)$ **h)** $1.8(x+1)(x+1)$

5. Expand and simplify.

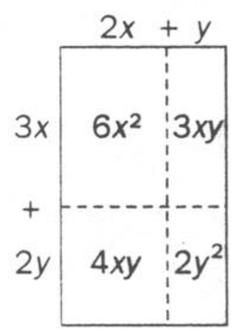
a) $(x+6)(x+4)+(x+2)(x+3)$ **b)** $(y-3)(y-1)-(y+2)(y-6)$
c) $(2x-3)(x+5)+(3x+4)(4x+1)$ **d)** $2(m+3)(m+5)+4(2m+3)$
e) $3(x-4)(x+3)-2(4x-1)$ **f)** $5(3t-4)(2t-1)-(6t-5)$
g) $2(3x+2)(3x+2)-3(2x-1)(2x-1)$ **h)** $12-2(3y-2)(3y+2)-(2y+5)(2y+5)$

Applications and Problem Solving

6. a) Verify that $(x+6)(x+2) \neq x^2+12$ by substituting 1 for x.
b) Expand $(x+6)(x+2)$ correctly.

B

7. a) Communication Explain how the diagram models the product $(2x+y)(3x+2y)$.
b) State the product in simplified form.



8. Expand and simplify.

a) $(3x+y)(x+4y)$ **b)** $(4a-b)(2a-5b)$ **c)** $(5m+2n)(4m-3n)$
d) $(4s-3t)(5s-6t)$ **e)** $(7a+8b)(a-b)$ **f)** $(-3a+4b)(2a+3b)$

9. Construction A square building of side x metres is extended by 10 m on one side and 5 m on the other side to form a rectangle.

a) Express the new area as the product of 2 binomials.

b) Evaluate the new area for $x = 20$.

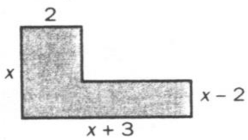
10. Diving Annie Pelletier won a bronze medal for Canada in women's springboard diving at the Summer Olympics in Atlanta. She dove from a springboard with dimensions that can be represented by the binomials $7x - 2$ and $x - 10$.

a) Multiply the binomials.

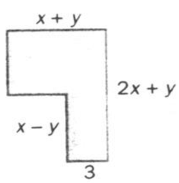
b) If x represents 70 cm, what was the area of the board, in square centimetres? in square metres?

11. Measurement Write and simplify an expression to represent the area of each figure.

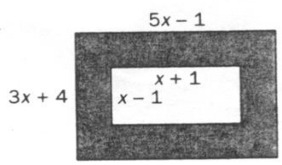
a)



b)

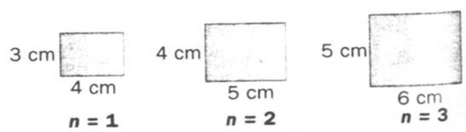


12. Measurement Write and simplify an expression to represent the area of the shaded region.



C

13. Pattern The diagrams show the first three rectangles in a pattern.



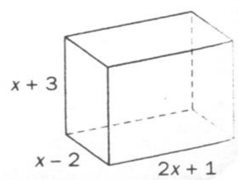
a) State the area of the 4th rectangle.

b) Write a product of two binomials to represent the area of the nth rectangle in terms of n.

c) Multiply the binomials from part b).

d) State the area of the 28th rectangle, in square centimetres.

14. Measurement The dimensions of a rectangular prism are represented by binomials, as shown.



a) Write, expand, and simplify an expression that represents the surface area of this prism.

b) If x represents 5 cm, what is the surface area, in square centimetres?

Answers

Practice 1. a) $x^2 + 6x + 5$ **b)** $x^2 + 7x + 12$ **c)** $a^2 + 8a + 16$ **d)** $y^2 + 11y + 30$ **e)** $x^2 - 7x + 12$ **f)** $a^2 - 6a + 8$ **g)** $b^2 - 6b + 5$ **h)** $y^2 - 18y + 81$ **i)** $x^2 - 3x - 18$ **j)** $c^2 - 6c - 16$ **k)** $t^2 - 100$ **l)** $q^2 + 3q - 10$ **2. a)** $c^2 - c - 12$ **b)** $x^2 - 3x - 10$ **c)** $y^2 + 4y - 12$ **d)** $a^2 + 4a - 45$ **e)** $x^2 - 9$ **f)** $b^2 + 3b - 70$ **g)** $y^2 - 9y - 36$ **h)** $x^2 - 6x - 7$ **i)** $28 + 3x - x^2$ **j)** $6 - 5y + y^2$ **k)** $x^2 + 14x + 49$ **l)** $b^2 - 64$ **3. a)** $2x^2 + 11x + 5$ **b)** $3y^2 + 7y + 2$ **c)** $2x^2 - 3x + 1$ **d)** $2a^2 - 11a + 15$ **e)** $5y^2 + 8y - 21$ **f)** $4x^2 - 17x - 15$ **g)** $9x^2 - 24x + 16$ **h)** $4 - 19t - 30t^2$ **i)** $9a^2 - 25$ **4. a)** $2x^2 + 16x + 30$ **b)** $4x^2 - 16x - 180$ **c)** $-a^2 - a + 6$ **d)** $10x^2 + 20x - 350$ **e)** $18x^2 - 21x + 6$ **f)** $-8y^2 + 22y + 6$ **g)** $0.5x^2 + x - 1.5$ **h)** $1.8x^2 + 3.6x + 1.8$ **5. a)** $2x^2 + 15x + 30$ **b)** 15 **c)** $14x^2 + 26x - 11$ **d)** $2m^2 + 24m + 42$ **e)** $3x^2 - 11x - 34$ **f)** $30t^2 - 61t + 25$ **g)** $6x^2 + 36x + 5$ **h)** $-22y^2 - 20y - 5$

Applications and Problem Solving 6. a) L.S. = 21, R.S. = 13 **b)** $x^2 + 8x + 12$ **7. a)** The length of the rectangle is $3x + 2y$. The width is $2x + y$. The area is $(3x + 2y)(2x + y)$. **b)** $6x^2 + 7xy + 2y^2$ **8. a)** $3x^2 + 13xy + 4y^2$ **b)** $8a^2 - 22ab + 5b^2$ **c)** $20m^2 - 7mn - 6n^2$ **d)** $20s^2 - 39st + 18t^2$ **e)** $7a^2 + ab - 8b^2$ **f)** $-6a^2 - ab + 12b^2$ **9. a)** $(x + 10)(x + 5)$ **b)** 750 m² **10. a)** $7x^2 - 72x + 20$ **b)** 29 280 cm²; 2.928 m² **11. a)** $x^2 + x - 2$ **b)** $x^2 + 3xy + 2y^2 + 3x - 3y$ **12.** $14x^2 + 17x - 3$ **13. a)** 42 cm² **b)** $(n + 2)(n + 3)$ **c)** $n^2 + 5n + 6$ **d)** 930 cm² **14. a)** $10x^2 + 10x - 10$ **b)** 290 cm²

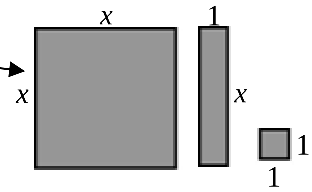
Unit 1 Review

General Review

1. Use ***algebra tiles*** to model each algebraic expression.

   a) $4x + 2$ **b)** $2x^2$

   c) $x^2 + 2x$ **d)** $2x^2 + x + 4$



2. One face of a cube has area 36 cm^2.

   a) What is the side length of the cube?

   b) Find the volume of the cube.

3. Evaluate.

   a) 5^3 **b)** 2^8

   c) -3^4 **d)** $(-2)^4$

   e) $(-1)^{10}$ **f)** $\left(\frac{2}{3}\right)^3$

4. Evaluate. Use the correct order of operations.

   a) $3^4 + 4^2$ **b)** $7^2 - 7$

   c) $9^2 \div 3^2$ **d)** $5 \times \left(\frac{2}{5}\right)^3$

   e) $(3^2 + 4^2)$ **f)** $(3 + 4)^2$

5. A scientist studying a type of bacteria notices that the population doubles every 30 minutes. The initial population is 500.

   a) Copy and complete the table.

Time (min)	Population
0	500
30	1000
60	
90	
120	

   b) Construct a graph of population versus time. Connect the points with a smooth curve.

6. Write as a single power. Then, evaluate.

   a) $8^5 \times 8^4 \div 8^7$

   b) $6^7 \div 6^5 \div 6$

   c) $(3^3)^4 \div 3^9$

   d) $\dfrac{(5^3)^4 \times 5^2}{5^{10}}$

   e) $2^7 \times 2^5 \div (2^2)^4$

   f) $[(-6)^3]^3 \div [(-6)^2]^4$

7. Simplify.

   a) b^6b^3 **b)** $g^2g^8 \div g^7$

   c) $(a^5)^3 \div (a^4)^2$ **d)** $m^5n \times m^2n^4$

   e) $\dfrac{p^7q^4}{p^3q^4}$ **f)** $\dfrac{8b^3d\left(4bd^2\right)}{2\left(2bd\right)^2}$

8. Identify the coefficient and the variable for each term.

   a) $7m$ **b)** $-3x^5$

   c) $\frac{3}{7}m^2n$ **d)** gh

9. Classify each expression as a monomial, binomial, trinomial or polynomial.

   a) $a^2 - 2a + 1$

   b) $2 - 3x^4 - 5x^2 + 4x$

   c) $6m^2n^5$

   d) $h^3 + 6$

   e) $12x$

   f) $4x^2 - 3y^2 + 8$

10. State the degree of each term.

    a) $-8b^4$

    b) $-x^4y^3$

    c) $\frac{3}{4}mn^2$

    d) $6r^6s$

11. What is the degree of each polynomial?

    a) $5a^4 + b^3$

    b) $7b^6$

    c) $2x^2 + 3x - 1$

    d) $8m^4 - m^2 + 2m$

12. Classify each pair of terms as like or unlike.

    a) $4a^2$ and $4a$

    b) $6x^3$ and $-x^3$

    c) $12p^4$ and $-p^4$

    d) $4a^2b^3$ and $6a^3b^2$

13. Simplify each expression.

a) $2b + 7g - 5b - 8g$

b) $3x + y^2 + 5y^2 - 7x$

c) $6q + u + 4u + q + u + 4u - u$

d) $10 - m^2 - 7 - m^2 + 4m^2$

e) $-3v + 2v + 6 - 3v - 9 - v$

f) $7 + h + h - 5 + 6h + 2 + 3h$

14. Simplify.

a) $(6k - 4) + (2k + 4)$

b) $(2a + 1) - (4a + 2)$

c) $(b - 6) - (2 - 5b) + (b + 4)$

d) $(g + 12) + (g - 7) - (2 - 3g)$

e) $(x^2 + 2x + 1) + (2x^2 + 4)$

f) $(2m^2 + m + 12) - (3m^2 + 4m - 6)$

15. The length of the Cheungs' backyard is double its width.

a) Write an expression for the perimeter of their back yard.

b) The width of their backyard is 9 m. What is its perimeter?

16. Expand.

a) $5(x + 3)$ **b)** $4(b + 2)$

c) $w(2w + 1)$ **d)** $q(q + 4)$

e) $3c(6 - 4c)$ **f)** $-p(2p - 1)$

g) $-5(a^2 - 4a - 2)$ **h)** $2d(d^2 - 3d - 1)$

17. Expand and simplify.

a) $3(x + 3) + 2(x + 1)$

b) $-4(m + 2) + 3(m - 7)$

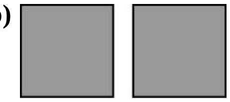
c) $-(d - 3) - 5(d + 2)$

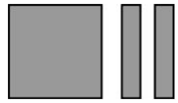
d) $5[b + 2(b + 1)]$

e) $-2[3(a + 3) - 4]$

f) $4[-2(4 - t) + 3t]$

Answers

1. a)  **b)** 

c)  **d)** 

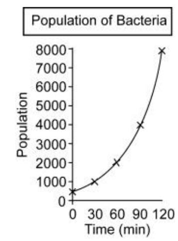
2. a) 6 cm **b)** 216 cm^3

3. a) 125 **b)** 256 **c)** –81 **d)** 16 **e)** 1 **f)** $\frac{8}{27}$

4. a) 97 **b)** 42 **c)** 9 **d)** $\frac{8}{25}$ **e)** 25 **f)** 49

5. a)

Time (min)	Population
0	500
30	1000
60	2000
90	4000
120	8000

b) 

6. a) 8^2; 64 **b)** 6^1; 6 **c)** 3^3; 27 **d)** 5^4; 625

e) 2^4; 16 **f)** $(-6)^1$; –6

7. a) b^9 **b)** g^3 **c)** a^7 **d)** m^7n^5 **e)** p^4 **f)** $4b^2d$

8. a) coefficient: 7; variable m

b) coefficient: –3; variable x^5

9. a) trinomial **b)** polynomial

c) monomial **d)** binomial

e) monomial **f)** trinomial

10.a) 4 **b)** 7 **c)** 3 **d)** 7

11.a) 4 **b)** 6 **c)** 2 **d)** 4

12.a) unlike **b)** like **c)** like **d)** unlike

13.a) $-3b - g$ **b)** $-4x + 6y^2$

c) $7q + 9u$ **d)** $3 + 2m^2$

e) $-5v - 3$ **f)** $4 + 11h$

14.a) $8k$ **b)** $-2a - 1$

c) $7b - 4$ **d)** $5g + 3$

e) $3x^2 + 2x + 5$ **f)** $-m^2 - 3m + 18$

15.a) $P = 6x$ **b)** 54 m

16.a) $5x + 15$ **b)** $4b + 8$

c) $2w^2 + w$ **d)** $q^2 + 4q$

e) $18c - 12c^2$ **f)** $-2p^2 + p$

g) $-5a^2 + 20a + 10$ **h)** $2d^3 - 6d^2 - 2d$

17.a) $5x + 11$ **b)** $-m - 29$

c) $-6d - 7$ **d)** $15b + 10$

e) $-6a - 10$ **f)** $-32 + 20t$

Problem Solving Review

1.

Meredith has a summer job at a fitness club. She earns a \$5 bonus for each student membership and a \$7 bonus for each adult membership she sells.

a) Write a polynomial expression that describes Meredith's total bonus.

b) Identify the variable and the coefficient of each term and explain what they mean.

c) How much will Meredith's bonus be if she sells 12 student memberships and 10 adult memberships?

2.

An arena charges \$25 for gold seats, \$18 for red seats, and \$15 for blue seats.

a) Write an expression that describes the total earnings from seat sales.

b) Identify the variable and the coefficient of each term and explain what they mean.

c) How much will the arena earn if it sells 100 gold seats, 200 blue seats, and 250 red seats?

3.

In a soccer league, teams receive 3 points for a win, 2 points for a loss, and 1 point for a tie.

a) Write an algebraic expression to represent a team's total points.

b) What variables did you choose? Identify what each variable represents.

c) The Falcons' record for the season was 5 wins, 2 losses, and 3 ties. Use your expression to find the Falcons' total points.

d) The 10-game season ended with the Falcons tied for second place with the same number of points as the Eagles. The Eagles had a different record than the Falcons. How is this possible?

4.

On a multiple-choice test, you earn 2 points for each correct answer and lose 1 point for each incorrect answer.

a) Write an expression for a student's total score.

b) Maria answered 15 questions correctly and 3 incorrectly. Find Maria's total score.

5.

Alberto is training for a triathlon, where athletes swim, cycle, and run. During his training program, he has found that he can swim at 1.2 km/h, cycle at 25 km/h, and run at 10 km/h. To estimate his time for an upcoming race, Alberto rearranges the formula distance = speed × time to find that: time $= \dfrac{\text{distance}}{\text{speed}}$.

a) Choose a variable to represent the distance travelled for each part of the race. For example, choose *s* for the swim.

b) Copy and complete the table. The first row is done for you.

Part of the Race	Speed (km/h)	Distance (km)	Time (h)
swim	1.2	s	$\frac{s}{1.2}$
cycle			
run			

c) Write a trinomial to model Alberto's total time.

d) A triathlon is advertised in Kingston. Participants have to swim 1.5 km, cycle 40 km, and run 10 km. Using your expression from part c), calculate how long it will take Alberto to finish the race.

e) Is your answer a reasonable estimate of Alberto's triathlon time? Explain.

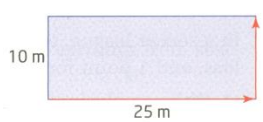
6.

Ashleigh can walk 2 m/s and swim 1 m/s. What is the quickest way for Ashleigh to get from one corner of her pool to the opposite corner?

a) Predict whether it is faster for Ashleigh to walk or swim.

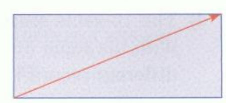
b) Ashleigh can walk at a speed of 2 m/s. The time, in seconds, for Ashleigh to walk is $\frac{w}{2}$, where w is the distance, in metres, she walks. Use this relationship to find the travel time if Ashleigh walks around the pool.

Path 1: Walk the entire distance.



c) Write a similar expression to represent the time taken for Ashleigh to swim a distance *s*. Her swimming speed is 1 m/s. Use this relationship to find the travel time if Ashleigh swims straight across.

Path 2: Swim the entire distance.

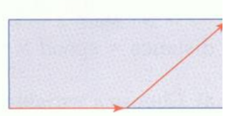


d) Which route is faster, and by how much?

7. Refer to question 6

a) Do you think it will be faster for Ashleigh to walk half the lengt and then swim? Explain your reasoning.

Path 3: Walk half the length, then swim.



b) Find the travel time for this path. Compare this with your answers to question 6.

c) Do you think this is the fastest possible path? Find the fastest path and the minimum time required to cross the pool, corner to opposite corner. Describe how you solved this.

This should be "note value" NOT "duration."
See http://en.wikipedia.org/wiki/Note_value .

8.

10. E. coli is a type of bacteria that lives in our intestines and is necessary for digestion. It doubles in population every 20 min. The initial population is 10.

a) Copy and complete the table. (The population of listeria doubles every 60 minutes

Time (min)	Population of Listeria	Population of E. Coli
0	800	
20		
40		
60		
80		
100		
120		

b) When will the population of E. coli overtake the population of Listeria?

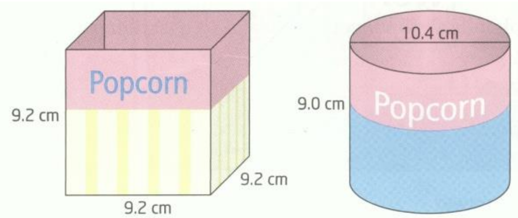
c) What population will the two cultures have when they are equal?

9.

The durations (lengths of time) of musical notes are related by powers of $\frac{1}{2}$, beginning with a whole note. Copy and complete the table.

Note	Symbol	Duration (in beats)	Power Form
whole	𝅝	1	
half	𝅗𝅥	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^1$
quarter	𝅘𝅥	$\frac{1}{4}$	$\left(\frac{1}{2}\right)^2$
eighth	𝅘𝅥𝅮		
sixteenth	𝅘𝅥𝅯		
thirty-second	𝅘𝅥𝅰		

10.



Mega-Box **Jumbo Drum**

Which container holds more popcorn? How much more? Assume that each container is filled just to the top. Round your answer to the nearest cubic centimetre.

11.

Refer to question 9. Look at the pattern in the last column. Extend this pattern backward to write the power form for a whole note. Does this answer make sense? Use a calculator to evaluate this power. Describe what you observe.

12.

Uranium-233 is another isotope that is used in nuclear power generation. 1 kg of U-233 can provide about the same amount of electrical power as 3 000 000 kg of coal. This number can be written in **scientific notation** as 3×10^6.

a) Another isotope of uranium, U-238, has a half-life of 4 500 000 000 years. Write this number in scientific notation.

b) What is the half-life of U-238, in seconds? Write your answer in scientific notation.

c) The number 6.022×10^{23} is a very important number in chemistry. It is called "one mole." One mole is the amount of a substance that contains as many atoms, molecules, ions, or other elementary units as the number of atoms in 12 g of carbon-12. Carbon-12 is the basic building block of living things. Write one mole in standard notation.

d) Describe any advantages you see to using scientific notation.

13.

Math Contest Determine the last digit of the number 3^{1234} when written in expanded form. Justify your answer.

14.

Math Contest If $3^x = 729$, the value of x is

A 3 **B** 5 **C** 6 **D** 7 **E** 8

15.

Math Contest Numbers are called perfect powers if they can be written in the form x^y for positive integer values of x and y. Find all perfect powers less than 1000.

16.

Math Contest x^x is always greater than y^y as long as $x > y$. For what whole-number values of x and y is $x^y > y^x$? Justify your answer.

17.

A computer repair technician charges \$50 per visit plus \$30/h for house calls.

a) Write an algebraic expression that describes the service charge for one household visit.

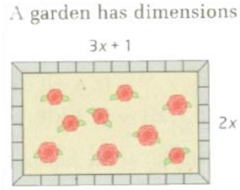
b) Use your expression to find the total service charge for a 2.5-h repair job.

c) Suppose all charges are doubled for evenings, weekends, and holidays. Write a simplified expression for these service charges.

d) Use your simplified expression from part c) to calculate the cost for a 2.5-h repair job on a holiday. Does this answer make sense? Explain.

18.

A garden has dimensions as shown.



a) Find a simplified expression for the perimeter.

b) Find a simplified expression for the area.

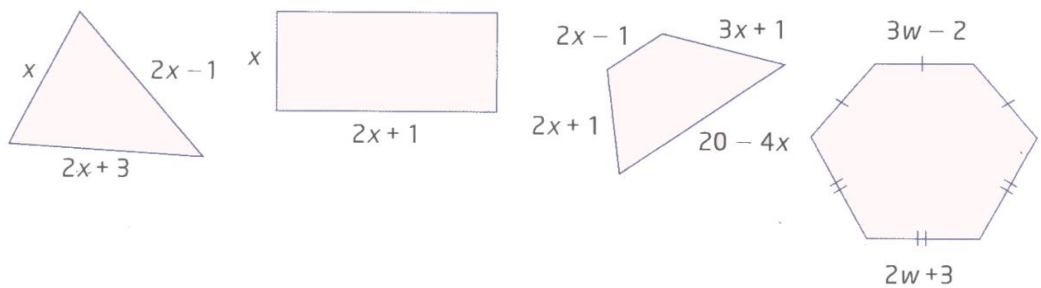
c) Repeat parts a) and b) if both the length and width are tripled.

d) Has this tripled the perimeter? Justify your answer.

e) Has this tripled the area? Justify your answer.

19.

a) Find a simplified expression for the perimeter of each figure. Use algebra tiles if you wish.



b) A rectangle has length $2x - 1$ and width $8 - 2x$. What is unusual about the perimeter?

c) For what value of x is the rectangle in part b) also a square?

Answers

1. (a) $5s + 7a$

(b) s = # student memberships sold
a = # adult memberships sold
5 = cost in \$ per student membership
7 = cost in \$ per adult membership

(c) \$130.00

2. (a) $25g + 18r + 15b$

(b) g = # gold tickets sold, r = # red tickets sold
b = # blue tickets sold
25 = cost in \$ per gold seat ticket
18 = cost in \$ per red seat ticket
15 = cost in \$ per blue seat ticket

(c) \$10,000.00

3. (a) $3w + 2l + t$

(b) w = # wins, l = # losses, t = # ties
3 = points per win, 2 = points per loss, 1 = points per tie

(c) 22

(d) There is more than one way to get 22 points. For example, 2 wins and 8 losses also results in 22 points in 10 games. (You should note that this point system is quite silly because a loss is more valuable than a tie. A team that is losing has no incentive to work hard to achieve a tie.)

4. (a) $2c - i$ (c = # correct answers, i = # incorrect answers)

(b) 27

5. (a) s = distance travelled swimming (km)
c = distance travelled cycling (km)
r = distance travelled running (km)

(b)

Part of the Race	Speed (km/h)	Distance (km)	Time (h)
swim	1.2	s	$\frac{s}{1.2}$
cycle	25	c	$c/25$
run	10	r	$r/10$

(c) $\dfrac{s}{1.2}+\dfrac{c}{25}+\dfrac{r}{10}$ (d) $\dfrac{1.5}{1.2}+\dfrac{40}{25}+\dfrac{10}{10}=3.85\text{ h}=3\text{ h, }51\text{ min}$

(e) The given speeds are reasonable because they are significantly lower than world record speeds. Therefore, the final answer seems reasonable.

6. (a) Ashleigh should take less time walking because her walking speed is twice her swimming speed but her walking distance is less than twice her swimming distance.

(b) 17.5 s

(c) 26.9 s

(d) walking is faster by 9.4 s

7. (a) It's possible that this route is faster because it is much shorter than path 1. Also, the swimming distance for this route is much shorter than for path 2.

(b) 22.3 s (which means this path is not faster)

(c) Path 1 is the fastest of the three routes. Calculus can be used to show that path 1 is the fastest route possible.

8. (a)

Time (min)	Population of Listeria	Population of E. Coli
0	800	10
20		20
40		40
60	1600	80
80		160
100		320
120	3200	640

(b) About 3 h, 10 min (190 minutes)
(c) About 7000

9.

Note	Symbol	~~Duration (in beats)~~	Power Form
whole	𝅝	1	
half	𝅗𝅥	$\frac{1}{2}$	$\left(\frac{1}{2}\right)^1$
quarter	♩	$\frac{1}{4}$	$\left(\frac{1}{2}\right)^2$
eighth	♪	$\frac{1}{8}$	$\left(\frac{1}{2}\right)^3$
sixteenth	𝅘𝅥𝅯	$\frac{1}{16}$	$\left(\frac{1}{2}\right)^4$
thirty-second	𝅘𝅥𝅰	$\frac{1}{32}$	$\left(\frac{1}{2}\right)^5$

10. Volume of Mega-Box = $(9.2)^3 \doteq 778.7 \text{ cm}^3$

Volume of Jumbo Drum = $\pi(5.2)^2(9) \doteq 764.6 \text{ cm}^3$

Mega-Box holds more popcorn (about 14.1 cm^3 more)

11. $\left(\frac{1}{2}\right)^0 = 1$. This makes sense according to the pattern that exists for the rest of the notes.

12. (a) 4.5×10^9

(b) $4.5 \times 10^9 \times 365 \times 24 \times 60 \times 60 = \mathbf{1.41912 \times 10^{17}}$

(c) 602,200,000,000,000,000,000,000
(six hundred and two sextillion, two hundred quintillion)

(d) Scientific notation is very useful for expressing long numbers containing long strings of zeroes. Such numbers, when written in scientific notation, are much easier to read and understand.

13. $3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$
$3^5 = 243$, $3^6 = 729$, $3^7 = 2187$, $3^8 = 6561$
$3^9 = 19683$, $3^{10} = 59049$, $3^{11} = 177147$, $3^{12} = 531441$

This pattern continues indefinitely.

Note that when the exponent is divisible by 4, the last digit is 1. Therefore, 3^{1232} must end in 1 because 1232 is divisible by 4. To continue the pattern, 3^{1233} must end in 3 and **3^{1234} must end in 9**.

14. C ($x = 6$)

15. $2^1 = 2$, $2^2 = 4$, $2^3 = 8$, $2^4 = 16$, $2^5 = 32$, $2^6 = 64$
$2^7 = 128$, $2^8 = 256$, $2^9 = 512$
$3^1 = 3$, $3^2 = 9$, $3^3 = 27$, $3^4 = 81$, $3^5 = 243$,
$3^6 = 729$
$4^1 = 4$, $4^2 = 16$, $4^3 = 64$, $4^4 = 256$
$5^1 = 5$, $5^2 = 25$, $5^3 = 125$, $5^4 = 625$
$6^1 = 6$, $6^2 = 36$, $6^3 = 216$
$7^1 = 7$, $7^2 = 49$, $7^3 = 243$
$8^1 = 8$, $8^2 = 64$, $8^3 = 512$
$9^1 = 9$, $9^2 = 81$, $9^3 = 729$
$10^1 = 10$, $10^2 = 100$

16. $2^1 > 1^2$, $3^1 > 1^3$, $4^1 > 1^4$, $5^1 > 1^5$, …, $x^1 > 1^x$ for $x > 1$.
$3^2 > 2^3$, $3^4 > 4^3$, $3^5 > 5^3$, $3^6 > 6^3$, $3^7 > 7^3$, …
There are infinitely many other examples. A general answer to this question can be obtained using logarithms. You will learn about logarithms in grades 11 and 12.

17. (a) $30t + 50$ (t = number of hours)

(b) \$125.00

(c) $2(30t + 50) = 60t + 100$

(d) \$250.00 (This answer makes sense because it is double the answer for (b)).

18. (a) $P = 2(2x + 3x + 1) = 2(5x + 1) = 10x + 2$

(b) $A = 2x(3x + 1) = 6x^2 + 2x$

(c) $P = 2(6x + 9x + 3) = 2(15x + 3) = 30x + 6$
$A = 6x(9x + 3) = 54x^2 + 18x$

(d) Tripling the length and width ***also*** tripled the perimeter because $3(10x + 2) = 30x + 6$.

(e) Tripling the length and width ***did not*** triple the area. The area is actually ***nine times greater*** because $9(6x^2 + 2x) = 54x^2 + 18x$.

19. (a) $P = 5x + 2$ $\quad P = 6x + 2$ $\quad P = 3x + 21$ $\quad P = 3(2w + 3) + 3(3w - 2) = 6w + 9 + 9w - 6 = 15w + 3$

(b) $P = 2(2x - 1 + 8 - 2x) = 2(7) = 14$ This is unusual because the perimeter is constant. No matter what the value of x is, the perimeter always turns out to be 14.

(c) The rectangle is a square if $2x - 1 = 8 - 2x$. This is true if $x = 2.25$.

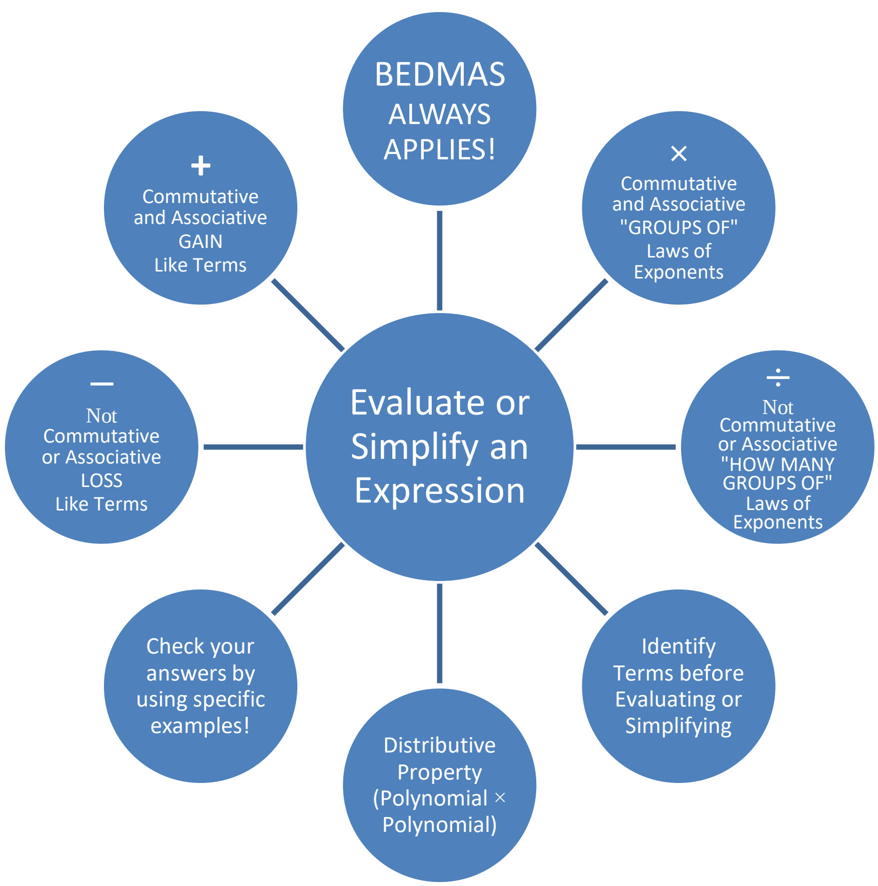
Unit 1 Reflection

The Basic Nature of Algebra: Arithmetic with Unknowns

Although algebraic expressions can appear to be intimidating, algebra is essentially a form of arithmetic that includes unknown values. Along with a few guiding principles, just about anyone can learn to understand and appreciate the power of algebra!

Guiding Principles

- **BEDMAS** applies to ***every mathematical expression***!
- **Identify Terms!** **e.g.** $\left(-2x^{-1}y^{3}\right)^{4}-\left(-2x^{-1}y^{4}\right)\left(5x^{-2}y^{-3}\right)-\dfrac{8x^{3}y\left(4xy^{2}\right)}{2\left(2xy\right)^{2}}$
- **Commutative Property** $\rightarrow$ $a+b=b+a$, $ab=ba$ Order doesn't Matter! (Not true for – and ÷)
- **Associative Property** $\rightarrow$ $(a+b)+c=a+(b+c)$, $(ab)c=a(bc)$ Brackets don't Matter! (Not true for – and ÷)
- **Adding and Subtracting** $\rightarrow$ Gains and Losses, Like Terms, Variable Part ***DOES NOT CHANGE***
- **Multiplying and Dividing** $\rightarrow$ "Groups of," "How many groups of," Laws of Exponents, Rearrange Factors (L to R)
- **Distributive Property** $\rightarrow$ Use to expand products of polynomials. Do not use to find products of monomials.
- **Use specific examples to check your thinking!**
  e.g. Is $x+x$ equivalent to x^2? **Check:** Let $x=3$. Then $x+x=3+3=6$ but $x^2=3^2=9$. Therefore, the two expressions cannot be equivalent.
- Always ***justify*** (give reasons for) your ***assertions*** (claims)!



An Important Reflection on Order of Operations: Does Order always Matter?

Overview of the Standard Order of Operations (Operator Precedence)

- The operations in ***ALL*** mathematical expressions must be performed in a specific ***standard*** order.
- This ensures that every mathematical expression evaluates to ***exactly one value***.
- We use the mnemonic "**BEDMAS**" to help us remember the standard order. This mnemonic has limitations, however, because it only includes the basic operations +, −, ×, ÷ and powers. As you expand your repertoire of mathematical operations ("functions"), it will be necessary to go beyond "BEDMAS" to understand the order in which the operations and functions must be applied.
- Parentheses ("brackets") are used to ***override*** the standard order.
  e.g. $2+3\times5=2+15=17$ ("×" before "+") $\quad\quad$ $(2+3)\times5=5(5)=25$ ("+" before "×")
- Most of the time, ***ORDER MATTERS***! However, there are ***exceptions***, some of which are listed below.

Exceptions: When Order doesn't Matter

1. **Commutative Property of "+" and "×":** $a+b=b+a$, $\quad ab=ba$
2. **Associative Property of "+" and "×":** $(a+b)+c=a+(b+c)$, $\quad (ab)c=a(bc)$
3. **Distributive Property:** $a(b+c)=ab+ac$ — Adding before multiplying gives the same result as multiplying before adding.
4. **Power of a Product:** $(ab)^c=a^cb^c$ — Multiplying before raising to the exponent gives the same result as raising to the exponent before multiplying.
5. **Quotient of a Product:** $\left(\dfrac{a}{b}\right)^c=\dfrac{a^c}{b^c}$ — Dividing before raising to the exponent gives the same result as raising to the exponent before dividing.

Exercises

Determine whether the following pairs of expressions are equivalent. If so, provide proof. Otherwise, give a counterexample.

Pairs of Expressions		*Equivalent?*	*Proof or Counterexample*
x^2+y^2	$(x+y)^2$	No	Let $x=1$ and $y=1$. Then $x^2+y^2=1^2+1^2=1+1=2$. However, $(x+y)^2=(1+1)^2=2^2=4$. Therefore, $x^2+y^2\neq(x+y)^2$.
x^2y^2	$(xy)^2$		
x^3+y^3	$(x+y)^3$		
$\sqrt{x+y}$	$\sqrt{x}+\sqrt{y}$		