Unit 2 – Equations: Powerful Problem Solving Tools

Introduction to Equations

What is an Expression

- An ***expression*** (or algebraic expression) is ***any*** mathematical calculation combining constants and/or variables using any valid mathematical operations. **e.g.** $-3x^2y+5abc-\dfrac{2xy^3z}{ab^2}-5\sqrt{z}$ is an (algebraic) expression
- An ***expression*** is a mathematical "***phrase***." (It's not a complete "sentence.")
  e.g. "Ten more than a number" → $x+10$

What is an Equation?

- An ***equation*** is a ***mathematical statement*** that asserts that ***two expressions are equal***.
- An ***equation*** is a (complete) ***mathematical "sentence."***
  e.g. "The sum of two consecutive numbers is 31." → $x+x+1=31$

What does an Equation Look Like?

Expression on the left-hand side (L.H.S.) = Expression on the right-hand side (R.H.S.)

An "equals" sign separates the two expressions.

Definitions

- A value is said to ***satisfy*** an equation if, when substituted for the unknown, the L.H.S. equals the R.H.S.
- Any value that satisfies an equation is called a ***solution*** or ***root*** of the equation.

Types of Equations

Equations that are Solved for the Unknown(s)	***Equations that Express a Relationship***	***Identities***
Zero or more (***but not all***) values of the unknown(s) satisfy these equations. Any value of the unknown that satisfies such an equation is called a ***solution*** or ***root***. **e.g.** What value(s) of x make(s) 2 times x plus 7 equal to 9? $2x+7=9$ $\therefore x=1$ Only ***one*** solution **e.g.** What value(s) of x make(s) x squared equal to 4? $x^2=4$ $\therefore x=2$ or $x=-2$ ***Two*** solutions! **e.g.** What value(s) of x make(s) x squared equal to −1? $x^2=-1$ There are ***NO*** solutions!	These equations ***describe*** how the values of ***two or more variables are related to each other***. **e.g.** The square of the hypotenuse of a right triangle is equal to the sum of the squares of the other two sides. $c^2=a^2+b^2$ **e.g.** The area of a triangle is half the product of the base and the height. $A=\dfrac{bh}{2}=\dfrac{1}{2}bh$ **e.g.** The volume of a rectangular prism is the product of the length, the width and the height. $V=lwh$	The L.H.S. and R.H.S. of the equation are ***equivalent***. ***All values*** of the unknown(s) satisfy the equation. **e.g.** 2 + 2 = 4 The expression on the L.H.S. is ***equivalent*** to the expression on the R.H.S. **e.g.** $x+x=2x$ The expression on the L.H.S. is ***equivalent*** to the expression on the R.H.S. For ***all values*** of x, the L.H.S. agrees with the R.H.S.

More on Identities

Equations that are ***Identities***	$2+2=4$	$x+x=2x$	$3x^2-5x-7x+2-3=3x^2-12x-1$	$a+2b+3c-a-2b-3c=0$
Equations that are ***NOT Identities***	$x+1=4$	$3x-7=-14$	$x^2+3x+2=0$	$x^2+2=0$

- An ***identity*** is an equation in which the expression on the L.H.S. is ***equivalent*** ("identical") to the expression on the R.H.S. In such equations, the L.H.S. equals the R.H.S. for ***all possible value(s)*** of the unknown(s).
- The ***expressions*** in equations that are ***not identities***, on the other hand, are ***not equivalent***. The L.H.S. equals the R.H.S. only for a specific value or specific values of the unknown. The values for which the L.H.S. equals the R.H.S. are called ***solutions*** of the equation. In addition, such values are said to ***satisfy*** the equation.

Exercise

1. State whether the given value(s) of the unknown(s) ***satisf(y/ies)*** the given equation. Show your work!

(a) $-3x-5=-4$, $x=-4$
(b) $-3x-5=-4$, $x=-3$
(c) $x^2+5x+6=0$, $x=-2$
(d) $x^2+5x+6=0$, $x=-3$
(e) $d=-7t+10$, $t=0$, $d=10$

2. Classify each of the following equations as ***identities*** (I) or ***equations that need to be solved*** (S).

(a) $x+3=4$ I / S **(b)** $(2x^3)(3x)^4=162x^7$ I / S **(c)** $3a+4a=7a$ I / S

(d) $2x-7=4$ I / S **(e)** $4-y=2$ I / S **(f)** $3g-4g=-g$ I / S

3. Classify each of the following equations as ***equations to be solved (S)***, ***equations that describe a relationship (R)*** or ***identities (I)***. State reasons for each choice.

(a) $x-5=-4$ S / R / I **Reasons:**	**(b)** $x-5x=-4x$ S / R / I **Reasons:**	**(c)** $-3xy(-5xy^3)=15x^2y^4$ S / R / I **Reasons:**
(d) $c^2=a^2+b^2$ S / R / I **Reasons:**	**(e)** $V=\frac{4}{3}\pi r^3$ S / R / I **Reasons:**	**(f)** $a^2+3a=-2$ S / R / I **Reasons:**
(g) $x^3+27=0$ S / R / I **Reasons:**	**(h)** $\frac{1}{2}(-3a-7)-\frac{3}{4}(2a)=-a+7$ S / R / I **Reasons:**	**(i)** $3xy(1-5xy^3)=3xy-15x^2y^4$ S / R / I **Reasons:**

4. Use ***trial and error*** to find solutions for each of the following equations:

(a) $x-5=-4$	**(b)** $-5x-7=-47$	**(c)** $-5(x-7)+3=-4x-15$
(d) $a^2+3a=-2$	**(e)** $x^3+27=0$	**(f)** $\frac{1}{2}(-3a-7)-\frac{3}{4}(2a)=-a+7$

5. Explain why ***trial and error*** is generally ***not*** a useful strategy when it comes to solving equations.

Techniques for Solving Equations

The Golden Rules of Solving Equations

1. Whatever operation is performed to one side of an equation ***must also be*** performed to the other side! This ensures that the resulting equation will have the ***same solution*** as the original equation.
2. The goal of solving an equation is to ***isolate*** the unknown (get it "by itself"). This is accomplished by ***undoing*** the operations performed to the unknown in the order ***opposite*** of **BEDMAS**.

Examples of One-Step and Two-Step Equations

(a)

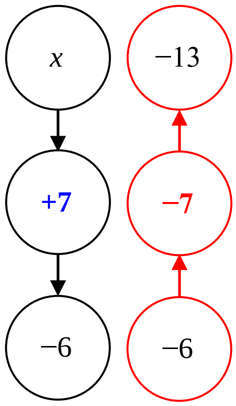
$x+7=-6$

$\therefore x+7-7=-6-7$

$\therefore x=-13$

In Words

Add 7 to a number and the result is −6. The number must be −13.



(b)

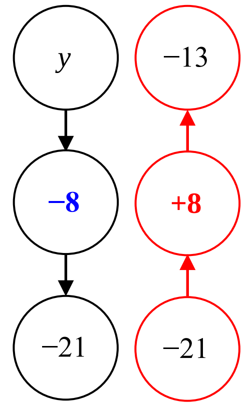
$y-8=-21$

$\therefore y-8+8=-21+8$

$\therefore y=-13$

In Words

Subtract 8 from a number and the result is −21. The number must be −13.



(c)

$-11a=-6$

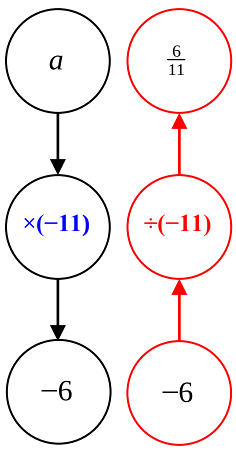
$\therefore \frac{-11a}{-11}=\frac{-6}{-11}$

$\therefore a=\frac{6}{11}$

In Words

Multiply a number by −11 and the result is −6.

The number must be $\frac{6}{11}$.



(d)

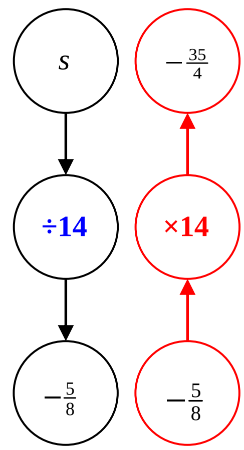
$\frac{s}{14}=-\frac{5}{8}$

$\therefore \frac{14}{1}\left(\frac{s}{14}\right)=\frac{14}{1}\left(-\frac{5}{8}\right)$

$\therefore s=-\frac{70}{8}=-\frac{35}{4}$

In Words

Divide a number by −14 and the result is $-\frac{5}{8}$. The number must be $-\frac{35}{4}$.



(e)

$-3x+7=-6$

$\therefore -3x+7-7=-6-7$

$\therefore -3x=-13$

$\therefore \frac{-3x}{-3}=\frac{-13}{-3}$

$\therefore x=\frac{13}{3}$

In Words

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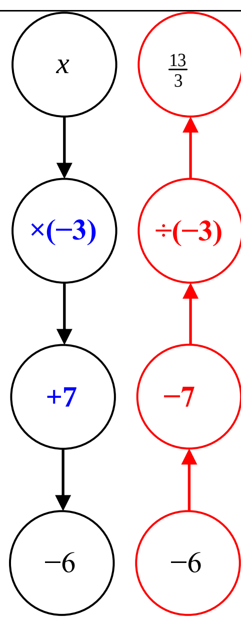

(f)

$4y-8=-21$

$\therefore 4y-8+8=-21+8$

$\therefore 4y=-13$

$\therefore \frac{4y}{4}=\frac{-13}{4}$

$\therefore y=\frac{-13}{4}$

In Words

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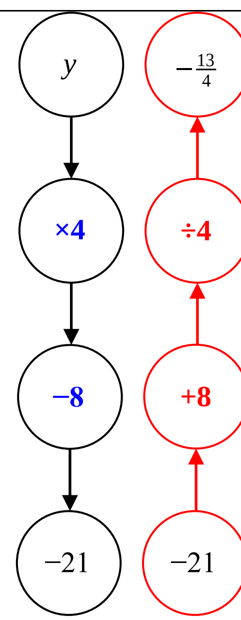
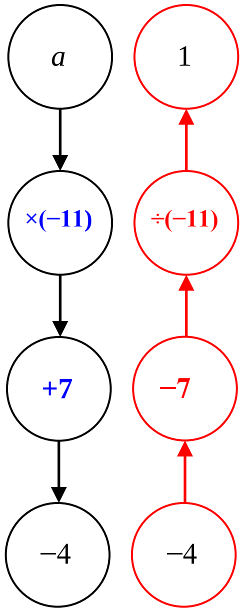


(g)

$$-11a+7=-4$$
$$\therefore -11a+7-7=-4-7$$
$$\therefore -11a=-11$$
$$\therefore \frac{-11a}{-11}=\frac{-11}{-11}$$
$$\therefore a=1$$



In Words

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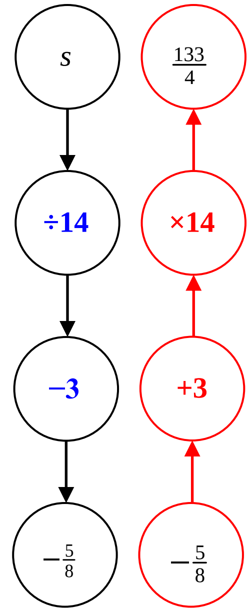
(h)

$$\frac{s}{14}-3=-\frac{5}{8}$$
$$\therefore \frac{s}{14}-3+3=-\frac{5}{8}+3$$
$$\therefore \frac{s}{14}=\frac{-5}{8}+\frac{3}{1}$$
$$\therefore \frac{s}{14}=\frac{-5}{8}+\frac{24}{8}$$
$$\therefore \frac{s}{14}=\frac{19}{8}$$
$$\therefore \frac{14}{1}\left(\frac{s}{14}\right)=\frac{14}{1}\left(\frac{19}{8}\right)$$
$$\therefore s=\frac{266}{8}=\frac{133}{4}$$



Checking your Solutions

Once you have obtained a ***tentative*** solution, it is a very good idea to check whether it ***satisfies*** both sides of the equation. Here are some examples.

(g) Tentative Solution: $a = 1$

L.H.S.	R.H.S.
$-11a+7$ $=-11(1)+7$ $=-11+7$ $=-4$	-4

Since L.H.S. = R.H.S., $a = 1$ is the correct solution.

(h) Tentative Solution: $s=\frac{133}{4}$

L.H.S.	R.H.S.
$\frac{s}{14}-3$ $=\frac{\left(\frac{133}{4}\right)}{14}-3$ $=\frac{133}{4}\div\frac{14}{1}-3$ $=\frac{133}{4}\times\frac{1}{14}-\frac{3}{1}$ $=\frac{133}{56}-\frac{168}{56}$ $=\frac{-35}{56}$ $=-\frac{35\div 7}{56\div 7}$ $=-\frac{5}{8}$	$-\frac{5}{8}$

Since L.H.S. = R.H.S., $s=\frac{133}{4}$ is the correct solution.

Homework

pp. 192-195 #6, 7, 8, 9, 10, 13, 15, 16, 17

Examples of Solving More Complicated Equations

(a)

$$-3(2x-7)+5x=4$$
$$\therefore -6x+21+5x=4$$
$$\therefore -6x+5x+21=4$$
$$\therefore -x+21=4$$
$$\therefore -x+21-21=4-21$$
$$\therefore -x=-17$$
$$\therefore x=17$$

(b)

$$-3(2x-7)+5x=-4x+5$$
$$\therefore -6x+21+5x=-4x+5$$
$$\therefore -6x+5x+21=-4x+5$$
$$\therefore -x+21=-4x+5$$
$$\therefore -x+21+4x=-4x+5+4x$$
$$\therefore 3x+21=5$$
$$\therefore 3x+21-21=5-21$$
$$\therefore 3x=-16$$
$$\therefore \frac{3x}{3}=\frac{-16}{3}$$
$$\therefore x=-\frac{16}{3}$$

(c)

$$-5(-4x-7)+5(-x-3)=4(2x-7)+3$$
$$\therefore 20x+35-5x-15=8x-28+3$$
$$\therefore 15x+20=8x-25$$
$$\therefore 15x+20-8x=8x-25-8x$$
$$\therefore 7x+20=-25$$
$$\therefore 7x+20-20=-25-20$$
$$\therefore 7x=-45$$
$$\therefore \frac{7x}{7}=\frac{-45}{7}$$
$$\therefore x=-\frac{45}{7}$$

(d)

$$-15(z-4)-(-15z-4)=4-3z$$
$$\therefore -15z+60+(15z+4)=4-3z$$
$$\therefore -15z+60+15z+4=4-3z$$
$$\therefore 64=4-3z$$
$$\therefore 64+3z=4-3z+3z$$
$$\therefore 64+3z=4$$
$$\therefore 64+3z-64=4-64$$
$$\therefore 3z=-60$$
$$\therefore \frac{3z}{3}=\frac{-60}{3}$$
$$\therefore z=-20$$

Exercises: Check the Solutions to (a) and (b) Above

(a) Tentative Solution: $x = 17$

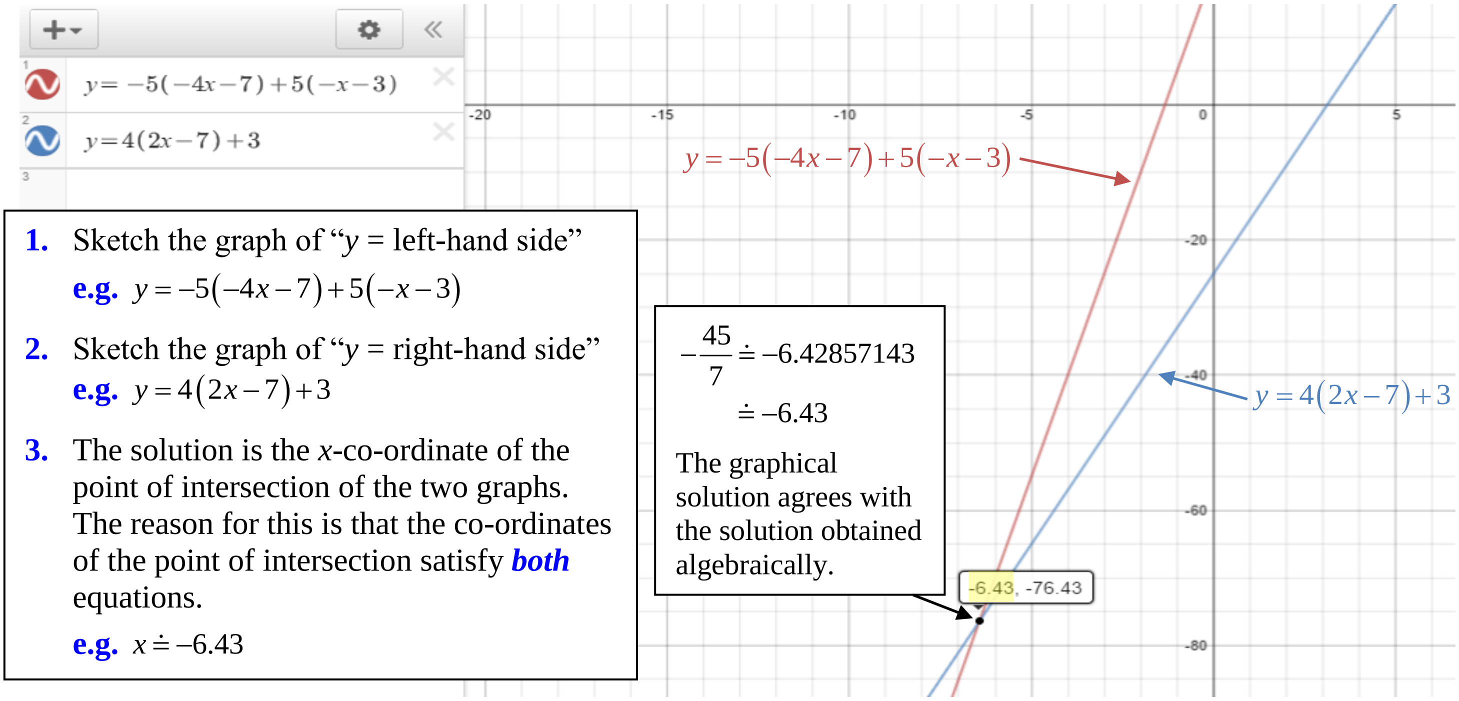
L.H.S.	R.H.S.

(b) Tentative Solution: $x=-\frac{16}{3}$

L.H.S.	R.H.S.

Using Graphing Technology to Check Solutions to Equations

On the previous page, we solved the equation $-5(-4x-7)+5(-x-3)=4(2x-7)+3$ and found the solution to be $x=-\frac{45}{7}$. Shown below is a graphical method that can be used to check the solution.



1. Sketch the graph of "y = left-hand side"
   e.g. $y=-5(-4x-7)+5(-x-3)$
2. Sketch the graph of "y = right-hand side"
   e.g. $y=4(2x-7)+3$
3. The solution is the x-co-ordinate of the point of intersection of the two graphs. The reason for this is that the co-ordinates of the point of intersection satisfy ***both*** equations.
   e.g. $x \doteq -6.43$

$-\frac{45}{7} \doteq -6.42857143$

$\doteq -6.43$

The graphical solution agrees with the solution obtained algebraically.

Summary

1. If possible, ***simplify*** both sides of the equation. **Remember!** Like Terms, Distributive Property, Add the Opposite.
2. If the ***variable*** (i.e. the unknown) ***appears on both sides*** of the equation, eliminate it from one side by performing the ***opposite*** operation to ***both sides*** of the equation.
3. If you have done everything correctly, by this stage you should have an equation with ***at most two*** operations to undo. ***Undo*** the operations in the order ***opposite*** of **BEDMAS**. Remember to perform the same operations to both sides!
4. Check the solution by substitution as well as with graphing technology.

Try this One!

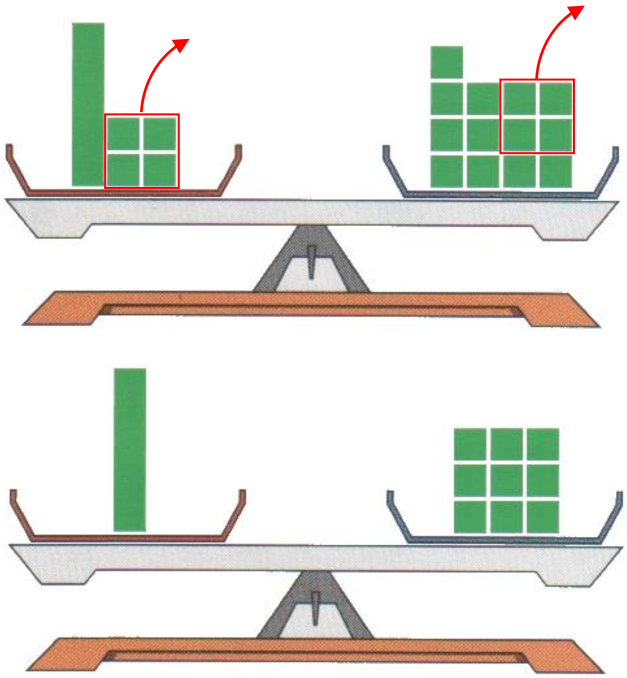
Solve the following equation. Then check your solution by substitution and with graphing technology.

$-13(-2z-3)-(15z+4)=-3-(4-3z)-7(3z-2)$

L.H.S.	R.H.S.

Why it Makes Sense to Perform the same Operation to Both Sides

If the same ***operation is performed to both sides*** of an equation, then ***equality still holds***. This can be compared to a balance scale as shown in the diagrams below. Balance is maintained as long as the weight on each side is the same. If weight is removed from or added to one side, then exactly the same weight must be removed from or added to the other side. Otherwise, one side will be heavier than the other will and the balance will be lost.



This picture represents the equation $x+4=13$. If a total of 4 units is removed from each side, the balance is maintained and the unknown is isolated.

Now that the unknown has been isolated, it's clear that the solution is $x=9$.

This rectangle represents x because its area is equal to x.

This square represents 1 because its area is equal to 1.

Homework

pp. 200 – 202, #5, 6, 7, 8, 9, 13, 14, 15

SOLVING EQUATIONS CONTAINING FRACTIONS

Equations that contain fractions require ***one additional step***. Otherwise, the rest of the process remains unchanged. The entire procedure is summarized below.

General Procedure for Solving Linear Equations

The equations that we have been solving in this unit are called ***linear equations*** because they involve polynomial expressions of degree at most one. In a later unit, you will better understand the significance of the word "linear."

1. If there are fractions in the equation, eliminate them by ***multiplying both sides of the equation by a common denominator of the fractions*** (preferably the lowest common denominator).
2. If possible, ***simplify*** both sides of the equation. **Remember!** Like Terms, Distributive Property, Add the Opposite.
3. If the ***variable*** (i.e. the unknown) ***appears on both sides*** of the equation, eliminate it from one side by performing the ***opposite*** operation to ***both sides*** of the equation.
4. If you have done everything correctly, by this stage you should have an equation with ***at most two*** operations to undo. ***Undo*** the operations in the order ***opposite*** of **BEDMAS**. Remember to perform the same operations to both sides!

Examples

(a)

$$\frac{y}{3}+\frac{1}{2}y=-\frac{5}{8}$$

The ***lowest common denominator*** is 24.
Therefore, both sides should be multiplied by 24.
3, 6, 9, 12, 15, 18, 21, 24, 27, 30, …
2, 4, 6, 8, 10, 12, 14, 16, 18, 20, 22, 24, …
8, 16, 24, …

This step can be omitted.

$$\therefore 24\left(\frac{y}{3}+\frac{1}{2}y\right)=24\left(-\frac{5}{8}\right)$$

$$\therefore \frac{24}{1}\left(\frac{y}{3}\right)+\frac{24}{1}\left(\frac{1}{2}\times\frac{y}{1}\right)=\frac{24}{1}\left(-\frac{5}{8}\right)$$

$$\therefore \frac{24}{3}\left(\frac{y}{1}\right)+\frac{24}{2}\left(\frac{y}{1}\right)=-\frac{120}{8}$$

$$\therefore 8y+12y=-15$$

$$\therefore 20y=-15$$

$$\therefore \frac{20y}{20}=\frac{-15}{20}$$

$$\therefore y=-\frac{3}{4}$$

(b)

$$\frac{1}{3}(y+2)+\frac{y-5}{6}=-\frac{4}{3}$$

$$\therefore 6\left(\frac{1}{3}(y+2)+\frac{y-5}{6}\right)=6\left(-\frac{4}{3}\right)$$

$$\therefore \frac{6}{1}\left(\frac{1}{3}\right)(y+2)+\frac{6}{1}\left(\frac{y-5}{6}\right)=\frac{6}{1}\left(-\frac{4}{3}\right)$$

$$\therefore \left(\frac{6}{3}\right)(y+2)+\frac{6}{6}\left(\frac{y-5}{1}\right)=-\frac{24}{3}$$

$$\therefore 2(y+2)+1(y-5)=-8$$

$$\therefore 2y+4+y-5=-8$$

$$\therefore 3y-1=-8$$

$$\therefore 3y-1+1=-8+1$$

$$\therefore 3y=-7$$

$$\therefore \frac{3y}{3}=\frac{-7}{3}$$

$$\therefore y=-\frac{7}{3}$$

Try This One!

$\frac{1}{4}(2y-7)+\frac{y-5}{6}=-3-(5y-8)$ (Answer: $y=\frac{91}{68}$)

Homework

pp. 208−209 #3, 4, 5, 6, 8, 9, 10, 11

Practice: Solving Equations Containing Fractions

1. Solve.

a) $\frac{c}{2}=7$

b) $\frac{n}{-3}=4$

c) $\frac{w}{-3}=-5$

d) $\frac{h}{6}=-3$

2. Find each root. (Remember that "root" means "solution.")

a) $2=\frac{1}{8}(s+7)$

b) $\frac{v+8}{5}=4$

c) $\frac{3}{4}(r-1)=6$

d) $\frac{u-8}{2}=-1$

e) $-\frac{1}{4}(z-5)=-1$

f) $\frac{2(e+5)}{3}=-2$

3. Find each root.

a) $\frac{b+3}{4}=\frac{b-1}{2}$

b) $\frac{d-1}{6}=\frac{d-3}{3}$

c) $\frac{1}{6}(z-4)=\frac{1}{2}(z-2)$

d) $\frac{x+4}{3}=\frac{x+6}{5}$

e) $\frac{3n+2}{8}=\frac{3n-2}{4}$

f) $\frac{1}{9}(2y-1)=\frac{1}{3}(y+1)$

4. Solve and check.

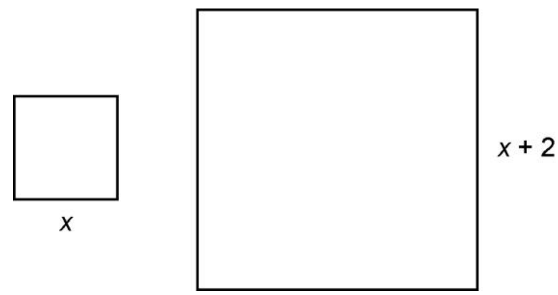
a) $k-3=\frac{k+3}{-5}$

b) $\frac{2z-3}{5}=3$

c) $\frac{1}{3}(9+g)=g+1$

d) $\frac{h+2}{3}=\frac{3h-2}{5}$

5. The perimeter of the small square is one-third the perimeter of the large square. What are the side lengths of the squares?



6. The height of a triangle is 2 cm less than its base. The area of the triangle is 24 cm^2. What are the measures of the base and height?

Answers

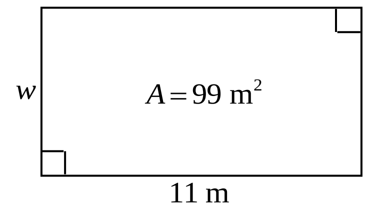
1. **a)** 14 **b)** –12 **c)** 15 **d)** –18
2. **a)** 9 **b)** 12 **c)** 9
d) 6 **e)** 9 **f)** –8
3. **a)** 5 **b)** 5 **c)** 1
d) –1 **e)** 2 **f)** –4

4. **a)** 2 **b)** 9 **c)** 3 **d)** 4
5. 1; 3
6. base: 8 cm; height: 6 cm

MANIPULATING (REARRANGING) EQUATIONS

Example 1

A rectangle has an area of 99 m^2 and a length of 11 m. What is its width?



Solution

1. Given: $A = 99$, $l = 11$ **Required to Find (RTF):** $w = ?$

2. Use the techniques that you have learned to ***solve*** for w ***in terms of*** A and l.

$$A = lw$$
$$\therefore \frac{A}{l} = \frac{lw}{l}$$
$$\therefore \frac{A}{l} = w$$
$$\therefore w = \frac{A}{l}$$

- ***Solve*** for w (i.e. "isolate" it, get it "by itself") ***in terms of*** A and l.
- Since w is multiplied by l, we can isolate w by performing the ***opposite*** operation (i.e. by dividing both sides by l).

3. Substitute the given information into the rearranged equation:

$$w = \frac{A}{l}$$
$$= \frac{99}{11}$$
$$= 9$$

The width of the rectangle is 9 m.

Example 2

The United States still uses the Fahrenheit temperature scale for weather reports and many other everyday purposes. The ***relationship*** between the Celsius and Fahrenheit temperature scales is given by the following equation:

$$F = \frac{9}{5}C + 32$$

This type of equation describes a ***relationship*** between the ***unknowns***.

Where F represents the temperature in degrees Fahrenheit and C represents the temperature in degrees Celsius.

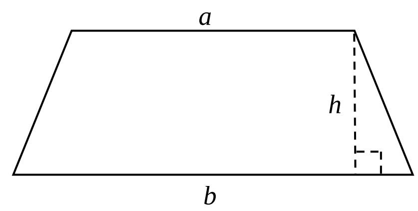
(a) Use the given equation to convert −40°C to °F. Is there anything strange about your result? Explain.

(b) ***Solve*** for C in terms of F. (Rearrange the equation to get C "by itself.")

(c) While travelling in the U.S., you read a weather report in an American newspaper. According to the report, the forecast high temperature for the day is 70°F. How would you dress for a high of 70°F?

Example 3

The area of a trapezoid is found by using the equation $A = \frac{h(a+b)}{2}$.



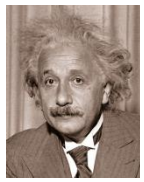
(a) ***Solve*** for h in terms of a, b and A.

(b) ***Solve*** for a in terms of h, b and A.

(c) ***Solve*** for b in terms of a, h and A.

Example 4

Einstein's famous equation, $E = mc^2$, describes the relationship between ***energy*** (E) and ***mass*** (m). In this equation, c represents the ***speed of light*** (approximately 300 000 km/s), a very important constant of nature.

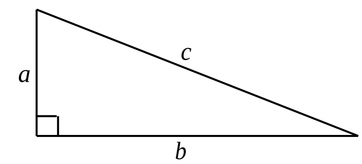


(a) ***Solve*** for m in terms of E and c.

(b) ***Solve*** for c in terms of E and m.

Example 5

The Pythagorean Theorem (also known as Pythagoras' Theorem) relates the lengths of the sides of a right triangle. According to this theorem, if *c* represents the length of the ***hypotenuse*** (the longest side of a right triangle) and *a* and *b* represent the lengths of the other two sides, then



$$c^2 = a^2 + b^2$$

(a) ***Solve*** for a^2.

(b) ***Solve*** for b^2.

(c) ***Solve*** for c.

Einsteinian Challenge!

Albert Einstein discovered that the universe can behave in strange and unexpected ways. For example, he discovered that the mass of an object is ***not constant***! According to Einstein's Special Theory of Relativity, the mass of an object ***depends*** on the velocity at which it is travelling! As counterintuitive as this startling result might seem, it has been confirmed by every experiment ever performed.

The ***relationship*** between the ***mass*** and ***velocity*** of an object is described by the equation given below. This equation is derived from revolutionary results that Einstein published in 1905. These results, along with their consequences, later came to be known as the *Special Theory of Relativity*. (The two groundbreaking papers published in 1905 that formed the foundation of Special Relativity are entitled *On the Electrodynamics of Moving Bodies* and *Does the Inertia of a Body Depend on its Energy-Content?*)

The Equation	***The Meaning of the Symbols***	***Example of Use***
$m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}}$	• v → The ***velocity*** of the object This is a ***variable*** quantity. • c → The ***speed of light*** This is a ***constant*** quantity. • m_0 → The ***rest mass*** (mass when $v = 0$) This is a ***constant*** quantity. • m → The ***mass*** (mass when $v > 0$) This is a ***variable*** quantity.	Calculate the mass of a 100.0 kg object moving at three-quarters the speed of light. ***Solution*** $m_0 = 100\text{ kg},\ c \doteq 299792\text{ km/s},\ m = ?$ $v = \dfrac{3}{4}c \doteq \dfrac{3}{4}(299792) = 224844\text{ km/s}$ $\therefore m = \dfrac{m_0}{\sqrt{1-\dfrac{v^2}{c^2}}} \doteq \dfrac{100.0}{\sqrt{1-\dfrac{224844^2}{299792^2}}} \doteq 151.2$

The Challenge

Solve for *v* in the equation given above (i.e. Einstein's equation that relates mass to velocity).

Summary

1. An equation that contains ***two or more variables*** and that ***is not an identity*** is often called a ***formula***. Such equations describe how the values of two or more variables are ***related*** to one another.

2. Such equations can be ***rearranged*** or ***manipulated*** by performing the ***same operation to both sides***.

3. The ***purpose*** of ***rearranging*** is to ***solve*** for one variable ***in terms of*** all the others.

Homework

pp. 214 #C3, 1, 2, 6, 8, 9, 10, 11, 12

Practice: Manipulating Equations (called "Modelling with Formulas" in the textbook)

1. The formula for area of a circle is $A = \pi r^2$ where r is the radius of the circle. Which is the formula rearranged to isolate r?

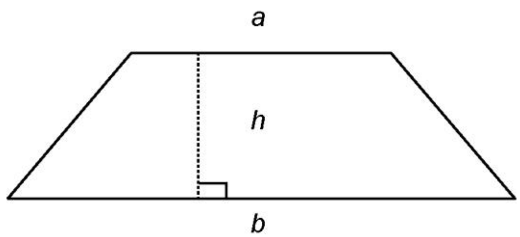
A $r = \dfrac{A}{\pi}$

B $r = \pi A$

C $r = \sqrt{\pi A}$

D $r = \sqrt{\dfrac{A}{\pi}}$

2. The formula for the area of a trapezoid is $A = \dfrac{h(a+b)}{2}$.



Which is the formula rearranged to isolate h?

A $h = \dfrac{2A}{a+b}$

B $h = 2A - (a + b)$

C $h = \dfrac{A(a+b)}{2}$

D $h = \dfrac{a+b}{2A}$

3. Rearrange each formula to isolate the variable indicated.

a) $P = 4s$ for s

b) $I = Prt$ for P

c) $A = \dfrac{bh}{2}$ for b

d) $P = 2(l + w)$ for l

e) $d = st$ for t

f) $V = \pi r^2 h$ for h

4. The approximate number of pounds, P, in a kilogram, K, is given by the formula $P = 2.2K$.

a) Christine's mass is 34 kg. Convert 34 kilograms to pounds.

b) Rearrange the formula to express K in terms of P.

c) Katherine weighs 78 pounds. Convert 78 pounds to kilograms.

5. The following shows Navpreet's first two steps in trying to solve for the variable a, given the formula for the area of a trapezoid. Use a flowchart diagram (like the ones on pages 6 – 7) to explain why the steps are incorrect.

$$A = \frac{h(a+b)}{2}$$

$$\therefore A - b = \frac{h(a+b)}{2} - b$$

$$\therefore A - b = \frac{ha}{2}$$

Answers

1. D

2. A

3. **a)** $s = \dfrac{P}{4}$ **b)** $P = \dfrac{I}{rt}$

c) $b = \dfrac{2A}{h}$ **d)** $l = \dfrac{P}{2} - w$

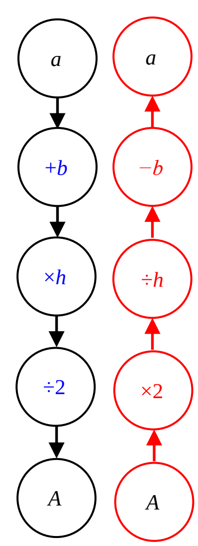
e) $t = \dfrac{d}{s}$ **f)** $h = \dfrac{V}{\pi r^2}$

4. **a)** 75 pounds

b) $K = \dfrac{P}{2.2}$

c) 35 kg

5. As shown in the flowchart at the right, Navpreet first should have multiplied both sides by 2. Subtracting b from both sides is actually the very last step needed to isolate a. Navpreet forgot that the operations must be undone in the order ***opposite*** of BEDMAS.



Using Equations to Solve Problems

Introduction: The Power of Equations

The main reason for learning to solve equations is that they prove to be powerful tools for solving problems. In this section, you will learn how to solve problems by constructing ***algebraic models***.

Example 1: The Power of Algebra!

Consider the following mathematical "magic trick." As you will soon discover, there is a simple mathematical explanation.

1. Think of any number. → x
2. Subtract 1 from the number. → $x-1$
3. Multiply the result by 3. → $3(x-1)$
4. Add 12 to the result. → $3(x-1)+12$
5. Divide the answer by 3. → $\dfrac{3(x-1)+12}{3}$
6. Add 5 to the answer. → $\dfrac{3(x-1)+12}{3}+5$
7. Subtract your original number. → $\dfrac{3(x-1)+12}{3}+5-x$
8. What is the result? → ?

First ask your subject to think of a number (preferably a small number to keep the calculations simple). After each step of the process, remind him/her to focus intently on the number obtained. Once all the steps have been performed, ask him/her to keep concentrating on the final result. Then, after pausing for a few moments to build suspense, confidently announce that you know the number that he/she is thinking about!

This "trick" works because the final answer ***always*** turns out to be ***eight***, regardless of the number that is selected at the outset! Why does this happen? A little algebra reveals the ***logic*** behind this so-called "trick." As shown below, it is merely a simple application of the rules that you have learned to simplify algebraic expressions.

$$\begin{aligned}&\frac{3(x-1)+12}{3}+5-x\\&=\frac{3(x-1)}{3}+\frac{12}{3}+5-x\\&=x-1+4+5-x\\&=x-x-1+4+5\\&=8\end{aligned}$$

The ***logic*** of this step is the same as in $\dfrac{3+12}{3}=\dfrac{3}{3}+\dfrac{12}{3}$

Example 2

The ***sum*** of ***three consecutive even numbers*** is 1122. What are the numbers?

Solution

(a) Construct an Algebraic Model

Since the numbers are unknown, we can use variables to represent their values. Only one variable is required because we know how ***the unknowns are related to one another***.

Let n represent the smallest number.

Number	***Representation***	***Explanation***
Smallest Even Number	n	Unknown
"Middle" Even Number	$n+2$	Two more than the unknown
Largest Even Number	$n+4$	Four more than the unknown

(b) Translate the Problem into an Equation

The **sum** of $\underbrace{\textbf{three consecutive even numbers}}_{n,\quad n+2,\quad n+4}$ **is** 1122.

$$n+(n+2)+(n+4)=1122$$

(c) Solve the Equation

$$n+(n+2)+(n+4)=1122$$
$$\therefore n+n+2+n+4=1122$$
$$\therefore 3n+6=1122$$
$$\therefore 3n+6-6=1122-6$$
$$\therefore 3n=1116$$
$$\therefore \frac{3n}{3}=\frac{1116}{3}$$
$$\therefore n=372$$

(d) State a Conclusion

Therefore, $n=372$, $n+2=374$ and $n+4=376$. The three consecutive even numbers must be 372, 374 and 376.

(e) Check the Solution

$372+374+376=1122$ ✓

Example 3

The ***perimeter*** of a rectangle is 34 m. If the length of the rectangle is ***double*** its width, find the dimensions of the rectangle.

Solution

(a) Construct an Algebraic Model

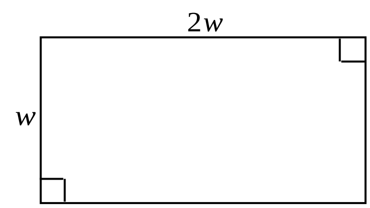
Let w represent the width of the rectangle. Since the ***length and width are related, only one variable is needed!***

Quantity	***Representation***	***Explanation***
Width	w	The width is unknown
Length	$2w$	The length is ***double*** the width.

(b) *Translate the Problem into an Equation*

The perimeter is 34.

$$2(w+2w)=34$$



(c) Solve the Equation

$$2(w+2w)=34$$
$$\therefore 2(3w)=34$$
$$\therefore 6w=34$$
$$\therefore \frac{6w}{6}=\frac{34}{6}$$
$$\therefore w=\frac{17}{3}$$

(d) State a Conclusion

Therefore, $w=\frac{17}{3}$ and $2w=2\left(\frac{17}{3}\right)=\frac{2}{1}\left(\frac{17}{3}\right)=\frac{34}{3}$.

The width is $\frac{17}{3}$ units and the length is $\frac{34}{3}$ units.

(e) Check the Solution

$$2\left(\frac{17}{3}+\frac{34}{3}\right)=2\left(\frac{51}{3}\right)=2(17)=34$$ ✓

Example 4

Amir is a carpeting salesperson. He is paid \$10.00 per hour worked ***plus*** 12¢ per square metre of carpeting sold.

(a) How much would Amir earn if he worked for 40 hours and sold 1500 m^2 of carpeting?

(b) How many square metres of carpeting must Amir sell to earn \$1000.00 in 20 hours?

Solution

In this example, it will not be possible to reduce the problem to a single variable. The reason for this is that the amount of carpeting sold is ***not directly related*** to the amount of time worked. Whenever business is slow, Amir could work many hours without selling much carpeting. During busy times, on the other hand, Amir might be able to sell large amounts of carpeting without having to work very long.

Quantity	***Representation***	***Algebraic Expression for Amount Earned***
Amount of Time Worked (in hours)	t	$10t \rightarrow$ Amount earned for working t hours
Amount of Carpeting Sold (in m^2)	c	$0.12c \rightarrow$ Amount earned for selling c m^2 of carpeting
Total Earnings (in \$)	E	$10t + 0.12c \rightarrow$ Total amount earned

(a) $t = 40$, $c = 1500$, $E = ?$

$$\begin{aligned} E &= 10t + 0.12c \\ &= 10(40) + 0.12(1500) \\ &= 400 + 180 \\ &= 580 \end{aligned}$$

Amir earns \$580.00 for working 40 hours and selling 1500 m^2 of carpeting.

(b) $t = 20$, $c = ?$, $E = 1000$

$$\begin{aligned} &E = 10t + 0.12c \\ &\therefore 1000 = 10(20) + 0.12c \\ &\therefore 1000 = 200 + 0.12c \\ &\therefore 1000 - 200 = 200 + 0.12c - 200 \\ &\therefore 800 = 0.12c \\ &\therefore \frac{800}{0.12} = \frac{0.12c}{0.12} \\ &\therefore 6666.67 \doteq c \end{aligned}$$

Amir must sell about 6700 m^2 of carpeting to earn \$1000.00 in twenty hours.

Summary – Using Equations to Solve Problems

1. Identify the **unknown(s)**.
Always remember to read the question carefully!

2. Construct an **algebraic model** by using a variable to represent one of the unknowns.

If there are **two or more unknowns** check whether the unknowns are related.
If the ***unknowns are related***, all unknowns should be expressed ***in terms of one variable***.
If the ***unknowns are not related*** (see example 4), more than one variable may be required.

3. **Translate** the problem into an equation.

4. **Solve** the equation.

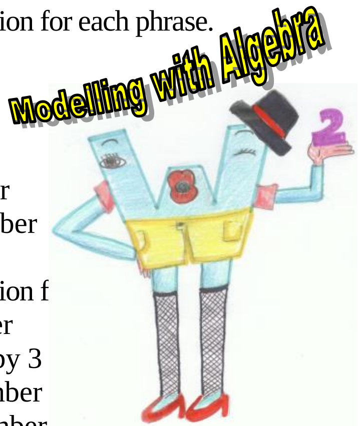
5. State a **conclusion**.

Homework

pp. 226−228 #1, 2, 3, 4, 7, 8, 9, 10, 11, 15

Practice: Modelling with Algebra ("Word Problems")

1. Write an algebraic expression for each phrase.
- **a)** double a number
- **b)** triple a number
- **c)** quadruple a number
- **d)** one half of a number
- **e)** one third of a number
- **f)** one quarter of a number



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2. Write an algebraic expression f
- **a)** 6 more than a number
- **b)** a number increased by 3
- **c)** 2 increased by a number
- **d)** 5 decreased by a number
- **e)** 7 less than a number
- **f)** a number decreased by 6

3. Write an algebraic expression for each phrase.
- **a)** 4 more than triple a number
- **b)** half a number, less 5
- **c)** quadruple a number decreased by 1
- **d)** 2 less than double a number

4. Write an equation for each phrase.
- **a)** triple a number is 18
- **b)** 7 more than a number is 11
- **c)** half a number is 10
- **d)** double a number, less 3 is 7
- **e)** 5 less than one third a number is 1
- **f)** 2 more than triple a number is 14

5. The sum of two consecutive integers is 47.
- **a)** Let x represent the lesser integer. Write an algebraic expression to represent the greater integer.
- **b)** Write an equation to represent the sum of the integers.
- **c)** Find the integers.

6. The sum of three consecutive odd integers is 57.
- **a)** Let x represent the least integer. Write an algebraic expression to represent each of the other integers.
- **b)** Write an equation to represent the sum of the integers.
- **c)** Find the integers.

7. Three consecutive even integers have a sum of 102.
- **a)** Write an algebraic expression to represent each integer.
- **b)** Write an equation to represent the sum of the integers.
- **c)** Find the integers.

8. Katherine is 2 years older than Christine. The sum of their ages is 16.
- **a)** Write an algebraic expression for each girl's age.
- **b)** Write an equation to represent the sum of their ages.
- **c)** How old is each girl?

9. The length of a rectangle is triple its width. The perimeter of the rectangle is 40 cm. What are the length and width?

Answers

1. **a)** $2x$ **b)** $3x$ **c)** $4x$
d) $\frac{1}{2}x$ **e)** $\frac{1}{3}x$ **f)** $\frac{1}{4}x$

2. **a)** $x+6$ **b)** $x+3$ **c)** $2+x$
d) $5-x$ **e)** $x-7$ **f)** $x-6$

3. **a)** $3x+4$ **b)** $\frac{1}{2}x-5$
c) $4x-1$ **d)** $2x-2$

4. **a)** $3x=18$ **b)** $x+7=11$
c) $\frac{x}{2}=10$ **d)** $2x-3=7$
e) $\frac{x}{3}-5=1$ **f)** $3x+2=14$

5. **a)** $x+1$
b) $x+x+1=47$
c) 23, 24

6. **a)** $x+2$, $x+4$
b) $x+x+2+x+4=57$
c) 17, 19, 21

7. **a)** x, $x+2$, $x+4$
b) $x+x+2+x+4=102$
c) 32, 34, 36

8. **a)** C, $C+2$
b) $16=C+C+2$
c) Katherine: 9; Christine: 7

9. 5 cm, 15 cm

Unit 2 Summary of Main Ideas

What is an Equation?

- An ***equation*** is a ***mathematical statement*** that asserts that ***two expressions are equal***.
- An ***equation*** is a (complete) ***mathematical "sentence."***
  e.g. "The sum of two consecutive numbers is 31." → $x + x + 1 = 31$
- Equations are very powerful problem solving tools.
- Equations come in three different varieties:

  Equations that are ***solved for the unknown*** (numeric solution): **e.g.** $3x - 7 = -23$

  Equations that ***describe a relationship*** between two or more unknowns: **e.g.** $c^2 = a^2 + b^2$

  Identities (equations that are satisfied by all possible values of the unknowns): **e.g.** $x^2 + x^2 = 2x^2$

What does an Equation Look Like?

Expression on the left-hand side (L.H.S.) = Expression on the right-hand side (R.H.S.)

The Golden Rules of Solving Equations

1. Whatever operation is performed to one side of an equation ***must also be*** performed to the other side! This ensures that the resulting equation will have the ***same solution*** as the original equation.
2. The goal of solving an equation is to ***isolate*** the unknown (get it "by itself"). This is accomplished by ***undoing*** the operations performed to the unknown in the order ***opposite*** of **BEDMAS**.

General Procedure for Solving Linear Equations

The equations that we have been solving in this unit are called ***linear equations*** because they involve polynomial expressions of degree at most one. In a later unit you will better understand the significance of the word "linear."

1. If there are fractions in the equation, eliminate them by ***multiplying both sides of the equation by a common denominator of the fractions*** (preferably the lowest common denominator).
2. If possible, ***simplify*** both sides of the equation. **Remember!** Like Terms, Distributive Property, Add the Opposite.
3. If the ***variable*** (i.e. the unknown) ***appears on both sides*** of the equation, eliminate it from one side by performing the ***opposite*** operation to ***both sides*** of the equation.
4. If you have done everything correctly, by this stage you should have an equation with ***at most two*** operations to undo. ***Undo*** the operations in the order ***opposite*** of **BEDMAS**. **Remember to perform the same operations to both sides!**

Rearranging Equations

1. An equation that contains ***two or more variables*** and that ***is not an identity*** is often called a ***formula***. Such equations describe how the values of two or more variables are ***related*** to one another.
2. Such equations can be ***rearranged*** or ***manipulated*** by performing the ***same operation to both sides***.
3. The ***purpose*** of ***rearranging*** is to ***solve*** for one variable ***in terms of*** all the others.

Using Equations to Solve Problems

1. Identify the **unknown(s)**.
   Always remember to read the question carefully!
2. Construct an **algebraic model** by using a variable to represent one of the unknowns.

   If there are **two or more unknowns** check whether the unknowns are related.
   - If the ***unknowns are related***, all unknowns should be expressed ***in terms of one variable***.
   - If the ***unknowns are not related*** (see Example 4 on page 19), more than one variable may be required.
3. **Translate** the problem into an equation.
4. **Solve** the equation.
5. State a **conclusion**.

Unit 2 Review Questions

4.1 Solve Simple Equations

1. Solve.

a) $5y = 35$ **b)** $b - 8 = -12$ **c)** $\frac{x}{4} = 7$ **d)** $h + 5 = 13$

2. Find each root.

a) $8m + 9 = -15$ **b)** $2p + 7 = 3$
c) $5 - 4k = -7$ **d)** $4 + 3c = -12$

3. Solve, then check.

a) $-2a = -22$ **b)** $3 - q = -5$
c) $\frac{1}{2}g = -9$ **d)** $7 - 6s = 19$

4. Greg is 42. He is 3 years older than Sue.

a) Write an equation relating Sue and Greg's ages.
b) How old is Sue?

4.2 Solve Multi-Step Equations

5. Solve.

a) $2m + 5m - 3 = 4$ **b)** $4b - 6 + b - 9 = 0$
c) $3x - x + 4 = 0$ **d)** $2k + 3 = 4k - 5$

6. Find the root of each equation.

a) $2 + (4h - 1) = 11 + 2h$ **b)** $8 - (2g + 3) = 3g - 5$
c) $2(d + 6) = 9(d - 1)$ **d)** $5(3r - 7) + r = 3(r - 3)$

7. Find each root, then check.

a) $4s + 3 - s = -6$ **b)** $p - 3 + 2p - 9 = 0$
c) $5 - (c + 3) = 4 + c$
d) $3(4d - 7) - 6 = 2(d + 2) - 1$

8. The perimeter of an isosceles triangle is 21 cm. The length of each equal side is triple the length of the base. Find the side lengths of the triangle.

4.3 Solve Equations Involving Fractions

9. Solve.

a) $\frac{t-6}{2} = 4$ **b)** $\frac{1}{3}(c + 2) = 1$
c) $\frac{4a+1}{3} = -5$ **d)** $\frac{2}{3}(s - 4) = 4$

10. Solve.

a) $\frac{d+4}{2} = \frac{3d}{4}$ **b)** $\frac{k-1}{2} = \frac{k+3}{4}$
c) $\frac{2}{3}(q - 3) = \frac{1}{4}(q + 7)$ **d)** $\frac{3c-1}{5} = \frac{4c+1}{9}$

4.4 Modelling With Formulas

11. Rearrange each formula to isolate the variable indicated.

a) $A = lw$ for l
b) $P = 2a + 2b$ for b
c) $y = mx$ for x
d) $l = w + 4$ for w
e) $P = 2a + b$ for b
f) $S = 2\pi r(r + h)$ for h

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4.5 Modelling With Algebra

12. Write an equation for each phrase.

a) 4 less than triple a number is 23
b) the sum of double a number and 6 is 16
c) half a number, less 3, is 8
d) the area decreased by 7 is 14
e) the sum of two consecutive integers is 49
f) the distance increased by 8 is 25

13. Together, Blackie and Jessie have a mass of 72 kg. Blackie's mass is 4 kg less than Jessie's mass. What is each dog's mass?

14. Chantal works at a music store. She earns \$8 per hour plus \$0.05 for each CD she sells. Tonight she is working a 5-h shift. How many CDs must Chantal sell to earn \$42?

Answers

1. **a)** 7 **b)** -4 **c)** 28 **d)** 8
2. **a)** -3 **b)** -2 **c)** 3 **d)** $\frac{-16}{3}$
3. **a)** 11 **b)** 8 **c)** -18 **d)** -2
4. **a)** $s + 3 = 42$ **b)** 39
5. **a)** 1 **b)** 3 **c)** -2 **d)** 4
6. **a)** 5 **b)** 2 **c)** 3 **d)** 2
7. **a)** -3 **b)** 4 **c)** -1 **d)** 3
8. 3 cm, 9 cm, 9 cm
9. **a)** 14 **b)** 1 **c)** -4 **d)** 10
10. **a)** 8 **b)** 5 **c)** 9 **d)** 2
11. **a)** $l = \frac{A}{w}$ **b)** $b = \frac{P-2a}{2}$ **c)** $x = \frac{y}{m}$
d) $w = l - 4$ **e)** $b = P - 2a$ **f)** $h = \frac{S}{2\pi r} - r$
12. **a)** $3x - 4 = 23$ **b)** $2x + 6 = 16$ **c)** $\frac{x}{2} - 3 = 8$
d) $A - 7 = 14$ **e)** $x + x + 1 = 49$ **f)** $d + 8 = 25$
13. Blackie: 34 kg; Jessie: 38 kg
14. 40

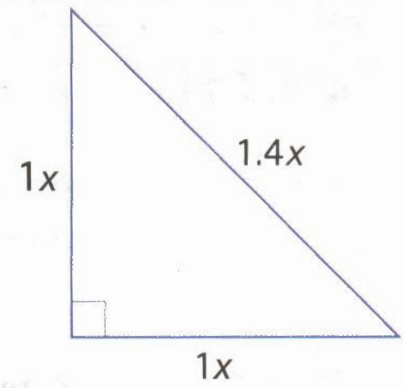
More Problems that can be Solved using Equations

1.

A family of right isosceles triangles has side lengths in the approximate ratio 1:1:1.4. A triangle belonging to this family has a perimeter of 50 cm.

a) Find the length of each side, to the nearest tenth of a centimetre.

b) Explain how you solved this.



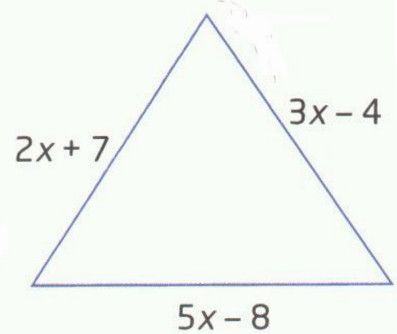
2.

Math Contest If $x = -4$ and $y = 3$ satisfy the equation $3x^2 + ky^2 = 24$, then which is the value of k?

A $\frac{1}{2}$ **B** $-\frac{1}{2}$ **C** 8 **D** $\frac{8}{3}$ **E** $-\frac{8}{3}$

3.

Math Contest Is there a value of x that makes this triangle equilateral? Explain your decision.



4.

Math Contest Diophantus of Alexandria was born around the year 200. He is known as the Father of Arithmetic. A puzzle about Diophantus is as follows:

"His boyhood lasted one sixth of his life. He married after one seventh more. His beard grew after one twelfth more and his son was born 5 years later. The son lived to half his father's final age, and the father died 4 years after the son."

How old was Diophantus when he died?

5.

The distance an accelerating object travels is related to its initial speed, v, its rate of acceleration, a, and time, t:

$$d = vt + \frac{1}{2}at^2$$

a) Rearrange this formula to isolate v.

b) An object travels 30 m while accelerating at a rate of 6 m/s^2 for 3 s. What was its initial speed?

6.

Math Contest The period (time for one complete swing back and forth) p, in seconds, of a pendulum is related to its length, L, in metres, by the formula $p = 2\pi\sqrt{\frac{L}{g}}$, where $g = 9.8$ m/s^2 is a constant. Solve this formula for L, and find the length needed for the pendulum to have a period of 1 s.

7.

Math Contest The escape velocity (speed needed to escape a planet's gravitational field), in metres per second, is given by $v = \sqrt{\frac{2GM}{r}}$.

$M = 5.98 \times 10^{24}$ kg is the mass of the Earth,
$G = 0.000\,000\,000\,066\,73$ (a constant), and r is the radius of the orbit. The average radius of Earth is 6.38×10^6 m.

a) Find the escape velocity for an Earth satellite in kilometres per second.

b) Solve the formula for M.

c) Find the mass of the planet Mars. Mars has a diameter of 6794 km. A Martian satellite requires an escape velocity of 5 km/s.

8.

Johnny is directly in front of Dougie, who is playing goalie, as shown.

Johnny is 2.8 m from both goal posts. He is also three times as far from Dougie as Dougie is from either post.

a) How wide is the net?

b) Describe how you solved this problem.

c) Discuss any assumptions you had to make.

9.

Math Contest The mass of a banana plus its peel is 360 g. The mass of the banana is four times the mass of the peel. What is the mass of the peel?

10.

Math Contest Given that $y = 4x + 1$ and $z = 5x - 3$, and the value of z is 7, what is the value of y?

A -2 **B** -9 **C** 2 **D** 9 **E** 29

11.

Johannes Kepler (1571–1630) was a German astronomer who noticed a pattern in the orbits of planets. The table shows data for the planets known when Kepler was alive.

Planet	Radius of Orbit (AU)*	Period of Orbit (Earth Days)
Mercury	0.389	87.77
Venus	0.724	224.70
Earth	1.0	365.25
Mars	1.524	686.98
Jupiter	5.200	4332.62
Saturn	9.150	10759.20

AU, or astronomical unit, is the mean distance from Earth to the Sun, 1.49×10^8 km.

a) Kepler conjectured that the square of the period divided by the cube of the radius is a constant. Copy the table. Add another column and compute the value of the square of the period divided by the cube of the radius for each planet. Then, find the mean of these values to find Kepler's constant.

b) Write a formula for the relationship that Kepler found. This is called Kepler's Third Law.

c) In 1781, William Herschel discovered the planet Uranus, which has a period of 30 588.70 days. Use Kepler's Third Law to determine the radius of Uranus's orbit.

d) In 1846, the planet Neptune was discovered. Neptune's orbital radius is 30 AU. Use Kepler's Third Law to find the orbital period of Neptune.

e) The planet Pluto has an orbital radius of 39.5 AU and a period of 90 588 days. Does Pluto satisfy Kepler's Third Law? Explain.

f) Investigate Kepler's other two laws of planetary motion. Write a brief report of your findings.

Answers

1. (a) 14.7 cm, 14.7 cm, 20.6 cm
(b) Solve the equation $x + x + 1.4x = 50$.

2. E

3. For the triangle to be equilateral, $2x + 7 = 5x - 8$ and $2x + 7 = 3x - 4$. Solving these equations yields different values of x ($x = 5$ and $x = 11$ respectively). Therefore, there is no value of x that makes the triangle equilateral.

4. Let x represent the age at which Diophantus died. Then solve the equation $\frac{1}{6}x+\frac{1}{7}x+\frac{1}{12}x+5+\frac{1}{2}x+4=x$ to obtain $x = 84$. Therefore, according to the riddle, Diophantus died at the age of 84.

5. (a) $v=\dfrac{2d-at^2}{2t}$
(b) 1 m/s

6. $L=\dfrac{p^2g}{4\pi^2}$, 0.248 m

7. (a) 11.18 km/s
(b) $M=\dfrac{rv^2}{2G}$
(c) 6.36×10^{23} kg

8. (a) 1.77 m
(b) Solve $x^2+(3x)^2=2.8^2$
(c) The goalie is standing right on the goal line, exactly midway between the goal posts.

9. 72 g

10. D

11. See p. 557 in the textbook