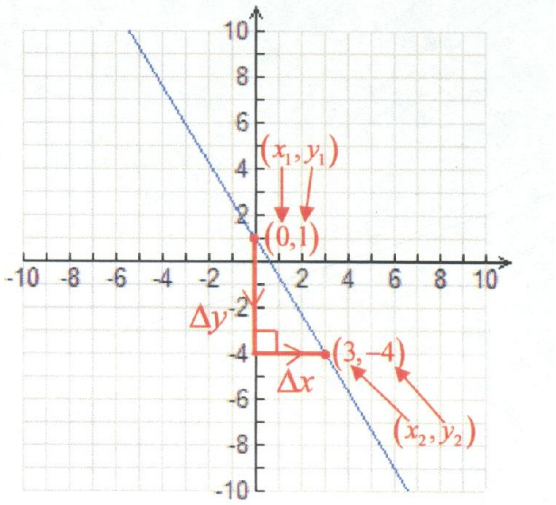
MPM1D9 Unit 4 – Linear Relations – Solutions

Solutions – Introduction – What you Should Already Know

Slopes, Intercepts and their MEANINGS

1. The ***slope*** of a line is a measure of the line's steepness.

   Slope also measures the rate of change of the dependent variable ***with respect to*** the independent variable. For example, in the graph shown at the right, slope $= m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-4-1}{3-0} = \frac{-5}{3}$. This ***means*** that for every ***increase*** in the independent variable ("x value") by 3 units, the dependent variable ("y value") decreases by 5 units.



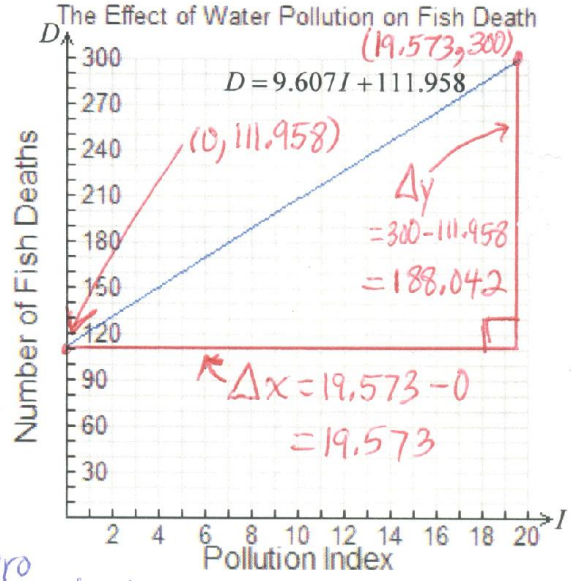
2. The ***y-intercept*** or ***vertical intercept*** is the y-co-ordinate of the point at which the graph intersects the y-axis.

   The ***meaning*** of the ***y-intercept*** is the value of the dependent variable when the ind. var. is 0

   In the graph shown above, the line intersects the y-axis at the point with co-ordinates (0, 1), which means that the ***y-intercept*** must be 1. The x-co-ordinate of the point at which graph intersects the y-axis must be zero because any point lying on the y-axis MUST have x-co-ordinate zero.

Applications of Slopes and Intercepts

1. Fisherman in the Finger Lakes Region have been recording the dead fish they encounter while fishing in the region. The Department of Environmental Conservation monitors the pollution index for the Finger Lakes Region. The mathematical model for the number of fish deaths "D" for a given pollution index "I" is $D = 9.607I + 111.958$.



   (a) Use the equation to identify the slope and the y-intercept of the given linear relation.

   slope $= m =$ 9.607 y-intercept $= b =$ 111.958

   (b) Mark the y-intercept on the graph. In addition, draw a right triangle that shows rise and run of the given line.

   (c) What is the ***meaning*** of the y-intercept?

   The y-intercept is the number of fish deaths for a pollution index of zero (i.e. # of fish deaths in completely unpolluted water).

   (d) What is the ***meaning*** of the slope? (**Hint:** slope = rate of change)

   The number of fish deaths increases by 9.607 (ie. 9 or 10) for every increase of 1 in the pollution index.

2. The *WeTalkALot* long distance company charges $5.00 each month for its special $0.05 per minute rate on long distance.

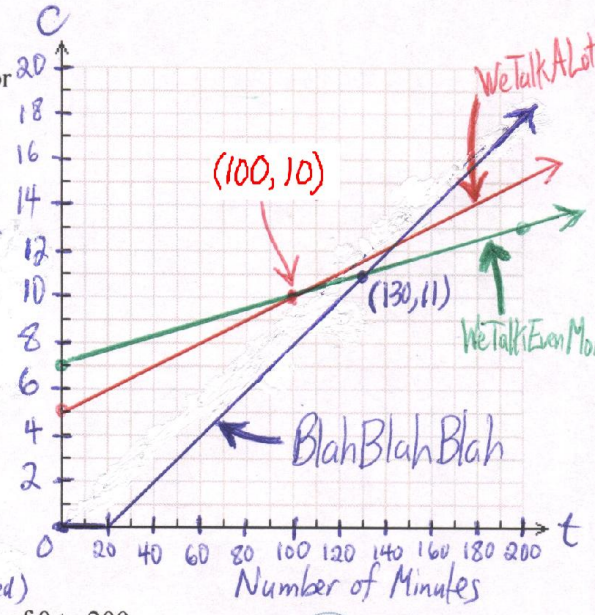
(a) Let C represent the total monthly long distance cost and let t represent the total number of minutes used for long distance calls. Write an equation that expresses C in terms of t.

$C = 0.05t + 5$

(b) State the slope, y-intercept and their meanings.

slope = m = 0.05 **Meaning:** $0.05 per minute

y-intercept = b = 5 **Meaning:** cost when number of minutes is zero (i.e. long distance not used)



(c) Use the provided grid to sketch a graph of C versus t. Use a scale of 0 to 200 minutes for the horizontal axis and scale of 0 to 20 dollars for the vertical axis. Don't forget to label the axes!

(d) The *WeTalkEvenMore* long distance company charges $7.00 per month for its special $0.03 per minute rate on long distance. Using the same grid given above, sketch a graph of C versus t for *WeTalkEvenMore*.

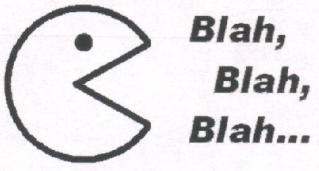
(e) If you have sketched both graphs correctly, you should find that they intersect (cross) at approximately the point with co-ordinates $(100,10)$. Explain the ***meaning*** of this point of intersection.

For both WeTalkALot and WeTalkEvenMore, the cost of 100 minutes of usage is $10.00.

(f) Under what circumstances is the *WeTalkALot* long distance plan a better deal? Under what circumstances is the *WeTalkEvenMore* plan a better deal?

Up to 100 minutes, WeTalkALot is a better deal than WeTalkEvenMore. For more than 100 minutes, WeTalkEvenMore is a better deal than WeTalkALot

(g) The *BlahBlahBlah* long distance company offers a unique plan. Each month, the first 20 minutes are free but thereafter, the calls cost $0.10 per minute. Using the same grid given above, sketch a graph of C versus t for *BlahBlahBlah*.



(h) If you have sketched the graph for *BlahBlahBlah* correctly, you will see that it has an ***x-intercept*** of 20. What is the meaning of the x-intercept?

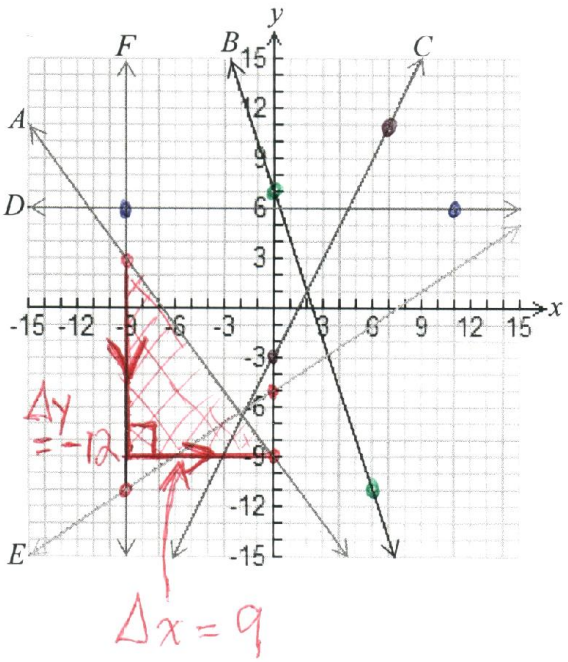
At 20 minutes, BlahBlahBlah begins charging for long distance calls.

(i) Under what circumstances is the *BlahBlahBlah* plan a better deal than either of the others?

BlahBlahBlah is a better deal than the others for up to 130 minutes.

Solutions – Equations of Lines Discovery Activity

Equations of Lines – Discovery Activity



1. Which of the lines shown at the right…
   - (a) …have ***positive*** slope? C, E
   - (b) …have ***negative*** slope? A, B
   - (c) …have ***zero*** slope? D
   - (d) …have ***undefined*** slope? F
2. Which line is steeper, …
   - (a) A or B? B
   - (b) E or C? C
   - (c) D or F? F
   - (d) A or C? C
   - (e) B or E? B

3. For each of the lines shown above, ***carefully select two points that lie on the line***. Then use those two points to calculate the slope of the line.
   Important Note: Make sure that the points that you choose lie ***where two grid lines intersect***.

   A. slope $= m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{3-(-9)}{-9-0} = \frac{12}{-9} = -\frac{4}{3}$

   B. slope $= m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{7-(-11)}{0-6} = \frac{18}{-6} = -3$

   C. slope $= m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{11-(-3)}{7-0} = \frac{14}{7} = 2$

   D. slope $= m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-6}{11-(-9)} = \frac{0}{20} = 0$

   E. slope $= m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-(-11)}{0-(-9)} = \frac{6}{9} = \frac{2}{3}$

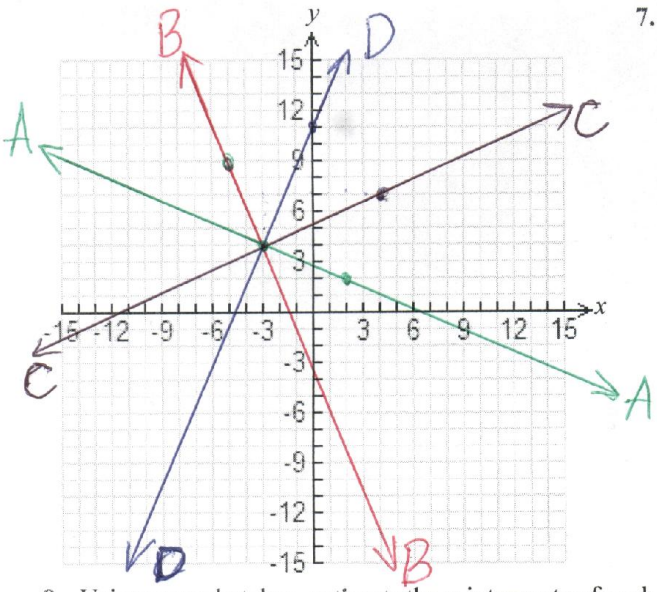
   F. slope $= m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6-(-12)}{-9-(-9)} = \frac{18}{0}$, which is undefined

4. For each of the lines shown above, ***identify the y-intercept***.
   - ***A.*** y-intercept $= b =$ -9
   - ***B.*** y-intercept $= b =$ 7
   - ***C.*** y-intercept $= b =$ -3
   - ***D.*** y-intercept $= b =$ 6
   - ***E.*** y-intercept $= b =$ -5
   - ***F.*** y-intercept $= b =$ undefined

5. For each of the lines shown above, ***write an equation in the form*** $y = mx + b$ (i.e. slope-y-intercept form).
   - ***A.*** $y = -\frac{4}{3}x - 9$
   - ***B.*** $y = 3x + 7$
   - ***C.*** $y = 2x - 3$
   - ***D.*** $y = 0x + 6 \Rightarrow y = 6$
   - ***E.*** $y = \frac{2}{3}x - 5$
   - ***F.*** cannot be written in $y = mx + b$ form

6. For each of the lines shown above, ***sketch a diagram showing both the rise and the run***. In each case, indicate the signs of Δx and Δy (i.e. whether the rise and run are positive or negative).

   Example shown for line A.



7. Use the provided grid to sketch lines passing through the point $(-3,4)$ and having…

   (a) …a slope of $-\frac{2}{5}$. Label this line *A*.

   (b) …a slope of $-\frac{5}{2}$. Label this line *B*.

   (c) …a slope of $\frac{3}{7}$. Label this line *C*.

   (d) …a slope of $\frac{7}{3}$. Label this line *D*.

8. Using your sketches, ***estimate*** the *y*-intercepts of each of the lines that you sketched in question 7.

   A. $b \doteq$ 2.8 *B.* $b \doteq$ -3.5 *C.* $b \doteq$ 5.1 *D.* $b \doteq$ 11

9. Using the example given below for line *A* as a model, calculate the *y*-intercepts of each of the lines in question 7. Then write an equation of each line in the form $y = mx + b$. (This is called the **slope-*y*-intercept equation** of a line.)

A. Since the slope of this line is known to be $-\frac{2}{5}$, the equation of the line must be of the form $y = -\frac{2}{5}x + b$. It's also given that the point $(-3,4)$ ***lies on the line***. Therefore, ***the co-ordinates of this point must satisfy the equation***. This means that when the values of x and y are substituted into the equation, the left-hand side ***must agree with*** the right-hand side.

$$4 = -\frac{2}{5}\left(\frac{-3}{1}\right) + b$$

$$\therefore 4 = -\frac{2}{5}\left(\frac{-3}{1}\right) + b$$

$$\therefore 4 = \frac{6}{5} + b$$

$$\therefore \frac{4}{1} - \frac{6}{5} = \frac{6}{5} + b - \frac{6}{5}$$

$$\therefore \frac{20}{5} - \frac{6}{5} = b$$

$$\therefore \frac{14}{5} = b$$

The slope-*y*-intercept equation of line *A* must be $y = -\frac{2}{5}x + \frac{14}{5}$.

B. $\because (-3,4)$ lies on the line and the slope is $-\frac{5}{2}$

$\therefore 4 = -\frac{5}{2}\left(\frac{-3}{1}\right) + b$

$\therefore 4 = \frac{15}{2} + b$

$\therefore 4 - \frac{15}{2} = \frac{15}{2} + b - \frac{15}{2}$

$\therefore \frac{8}{2} - \frac{15}{2} = b$

$\therefore b = -\frac{7}{2}$

$\therefore$ the slope-*y*-intercept equation of line B is $y = -\frac{5}{2}x - \frac{7}{2}$.

C. $\because (-3,4)$ lies on the line and the slope is $\frac{3}{7}$

$\therefore 4 = \frac{3}{7}\left(\frac{-3}{1}\right) + b$

$\therefore 4 = -\frac{9}{7} + b$

$\therefore \frac{4}{1} + \frac{9}{7} = -\frac{9}{7} + b + \frac{9}{7}$

$\therefore \frac{28}{7} + \frac{9}{7} = b$

$\therefore b = \frac{37}{7}$

$\therefore$ the slope-*y*-intercept equation of line C is $y = \frac{3}{7}x + \frac{37}{7}$

D. $\because (-3,4)$ lies on the line and the slope is $\frac{7}{3}$

$\therefore 4 = \frac{7}{3}\left(\frac{-3}{1}\right) + b$

$\therefore 4 = -7 + b$

$\therefore 4 + 7 = -7 + b + 7$

$\therefore 11 = b$

$\therefore$ the slope-*y*-intercept equation of the line is $y = \frac{7}{3}x + 11$

10. Carefully check your answers to questions 8 and 9. Summarize your results in the following table. If your answers to question 8 ***do not agree*** with your answers to question 9, then find out what went wrong and correct your mistakes!

8A. $b \doteq$ 2.8	8B. $b \doteq$ −3.5	8C. $b \doteq$ 5.1	8D. $b \doteq$ 11
9A. $b =$ $\frac{14}{5} = 2.8$	9B. $b =$ $-\frac{7}{2} = -3.5$	9C. $b =$ $\frac{37}{7} \doteq 5.3$	9D. $b =$ 11
Answers Agree? (Yes / No)	Answers Agree? (Yes / No)	Answers Agree? (Yes / No)	Answers Agree? (Yes / No)

11. Consider the linear relation with slope-y-intercept equation $y = -\frac{3}{2}x - \frac{7}{2}$.

(a) Describe the relation in words. Specifically, what does the equation tell you about the ***relationship*** between the x-co-ordinate and the y-co-ordinate of any point that lies on the line?

The y-co-ordinate is obtained by multiplying the x-co-ordinate by $-\frac{3}{2}$ and then subtracting $\frac{7}{2}$. In addition, the slope is $-\frac{3}{2}$ and the y-intercept is $-\frac{7}{2}$

(b) Complete the following table of values:

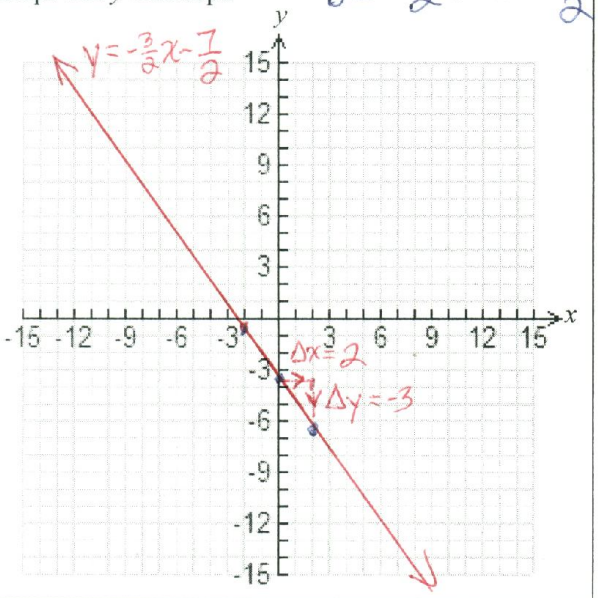
x	$y = -\frac{3}{2}x - \frac{7}{2}$
−5	$-\frac{3}{2}(\frac{-5}{1}) - \frac{7}{2} = \frac{15}{2} - \frac{7}{2} = \frac{8}{2} = 4$; (−5, 4)
−3	$-\frac{3}{2}(\frac{-3}{1}) - \frac{7}{2} = \frac{9}{2} - \frac{7}{2} = 1$; (−3, 1)
−1	$-\frac{3}{2}(\frac{-1}{1}) - \frac{7}{2} = \frac{3}{2} - \frac{7}{2} = \frac{-4}{2} = -2$; (−1, −2)
0	$-\frac{7}{2}$; $(0, -\frac{7}{2})$
1	−5 ; (1, −5)
3	−8 ; (3, −8)
5	−11 ; (5, −11)

co-ordinates

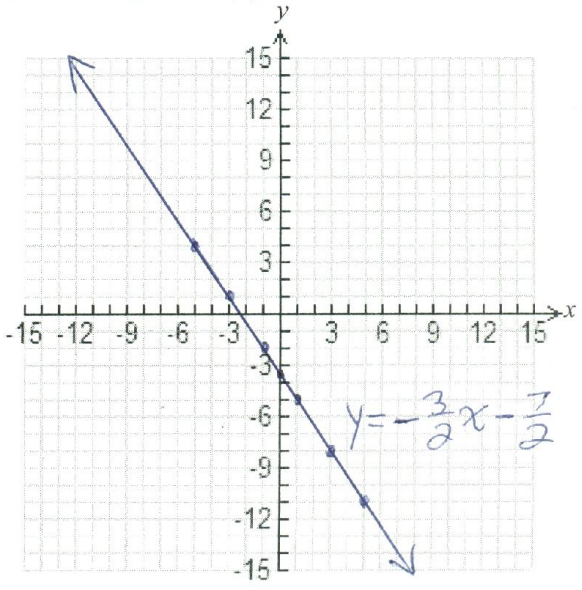
(c) Sketch the graph of $y = -\frac{3}{2}x - \frac{7}{2}$ using ***only*** the slope and y-intercept.

$b = -\frac{7}{2}$, $m = -\frac{3}{2}$

$m = -\frac{3}{2} = \frac{-3}{2} = \frac{3}{-2} = \frac{\text{rise}}{\text{run}}$



(d) Sketch the graph of $y = -\frac{3}{2}x - \frac{7}{2}$ using ***only*** the table of values from (b).



(e) Check carefully to ensure that the graphs in (c) and (d) are identical. If they are, then check with some classmates to see if your graphs agree.

If all the graphs agree, then they are probably correct. If any do not agree, then check your work and correct any mistakes.

(f) Summarize what you have learned from exercises 7 to 11.

- How to calculate the y-intercept when the slope and a point are known
- How to check for agreement of answers
- How to plot line using a table of values
- How to plot a line using slope and y-intercept

12. In this question you will explore various forms of equations for linear relations. The forms that you need to know are summarized in the table given below.

Slope-y-intercept Form	*Standard Form*	*"Modified" Standard Form*
$y = mx + b$ m and b are ***constants*** representing the ***slope*** and ***y-intercept*** respectively **e.g.** $y = -3x - 5$ slope = $m = -3$, y-intercept = $b = -5$	$Ax + By + C = 0$ A, B and C are ***constants*** that ***do not by themselves*** represent geometric features of the graph **e.g.** $2x - 5y - 3 = 0$ $A = 2$, $B = -5$, $b = -3$	$Ax + By = C$ A, B and C are ***constants*** that ***do not by themselves*** represent geometric features of the graph **e.g.** $2x - 5y = 3$ $A = 2$, $B = -5$, $b = 3$
Advantage Very easy to sketch the graph. **Disadvantage** Cannot be used with lines that have an undefined slope.	**Advantage** Can be used even if slope is undefined. **Disadvantage** More difficult to sketch the graph.	**Advantage** Can be used even if slope is undefined. **Disadvantage** More difficult to sketch the graph.

(a) Use your knowledge of rearranging equations to write $y = -\frac{3}{2}x - \frac{7}{2}$ in standard form. (**Hint:** Eliminate the fractions first!)

slope-y-intercept form $y = mx + b$

LCD is 2

$2y = \frac{2}{1}\left(-\frac{3}{2}x\right) - \frac{2}{1}\left(\frac{7}{2}\right)$

$\therefore 2y = -3x - 7$

$\therefore 2y + 3x + 7 = -3x - 7 + 3x + 7$

$\therefore 3x + 2y + 7 = 0$ Standard Form $Ax + By + C = 0$

(b) Check your answer to (a) by completing the following table of values.

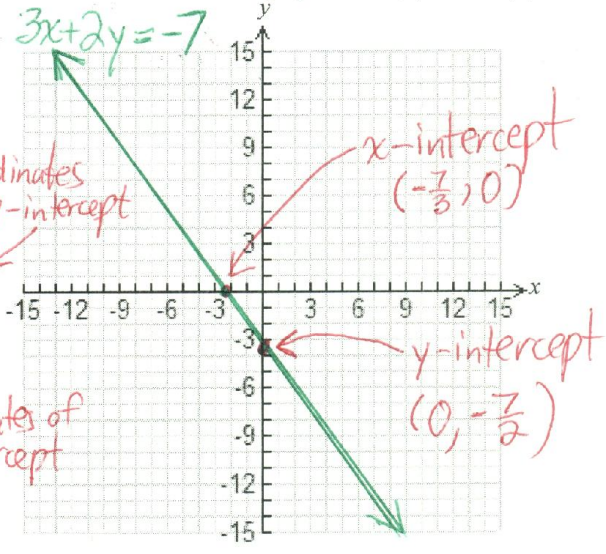
x	y (Calculated using $y = -\frac{3}{2}x - \frac{7}{2}$)	y (Calculated using the equation in standard form obtained in (a))
−4	5/2 (−3)	$3(-4) + 2y + 7 = 0 \rightarrow y = \frac{5}{2}$
−2	−1/2 (−3)	$3(-2) + 2y + 7 = 0 \rightarrow y = -\frac{1}{2}$
0	−7/2 (−3)	$3(0) + 2y + 7 = 0 \rightarrow y = -\frac{7}{2}$
2	−13/2 (−3)	$3(2) + 2y + 7 = 0 \rightarrow y = -\frac{13}{2}$
4	−19/2 (−3)	$3(4) + 2y + 7 = 0 \rightarrow y = -\frac{19}{2}$

(c) By now you should have the correct answer to (b). Rewrite the equation in the form $Ax + By = C$. Then use the equation to complete the following table. Show all your work!

$3x + 2y = -7$

x	y
0	$3(0) + 2y = -7$ $\therefore 2y = -7$ $\therefore \frac{2y}{2} = \frac{-7}{2}$ $\therefore y = -\frac{7}{2}$ $\left(0, -\frac{7}{2}\right)$ co-ordinates of y-intercept
$3x + 2(0) = -7$ $\therefore 3x = -7$ $\therefore \frac{3x}{3} = -\frac{7}{3}$ $\therefore x = -\frac{7}{3}$	0 $\left(-\frac{7}{3}, 0\right)$ co-ordinates of x-intercept

(d) Use the table in (a) to sketch the graph. Your graph must be identical to the graphs in 11(c) and 11(d).



(e) Use the first two columns of the table in (b) to explain why the relation ***must be*** linear.

The first differences are constant (all are equal to $-\frac{6}{2} = -3$). ∴

Solutions – Review of Analytic Geometry and Relations

1. For a taxi ride, a Toronto taxi company charges \$5.00 plus \$1.50 per kilometre travelled.

(a) Complete the following table of values:

d = distance (km), C = cost (\$)

d	C	ΔC (1st differences)
0	5	–
10	20	15
20	35	15
30	50	15
40	65	15
50	80	15

(b) Is this relation an example of direct variation or partial variation? Explain.

When $d = 0$, $C = 5$. Therefore, the line ***does not pass*** through the origin, which means that the variation must be ***partial***.

(c) Explain why the relation between C and d must be linear. In addition, state the slope and the y-intercept.

The relation must be linear because the first differences are CONSTANT

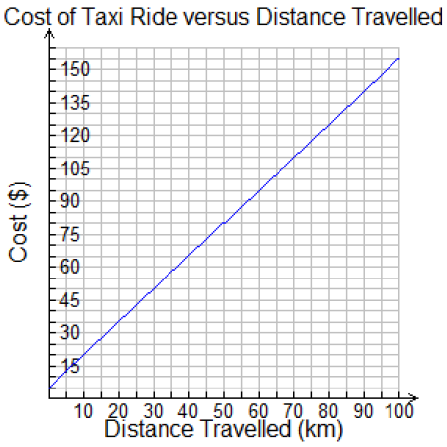
$$m = \frac{\Delta C}{\Delta d} = \frac{20-5}{10-0} = \frac{15}{10} = 1.5$$

$b = 5$ This follows from the fact that when $d = 0$, $C = 5$. (See table.)

(d) Which variable is the dependent variable? Explain.

The dependent variable is C because the cost depends on the distance travelled.

(e) Graph the relation.



(f) Write an equation, in the form $y = mx + b$, that relates C to d.

$$C = 1.5d + 5$$

(g) Interpret the slope as a rate of change.

The slope is 1.5. This means that the cost is \$1.50 per kilometre.

(h) Interpret the y-intercept as an initial value.

$b = 5$
This means that the taxi meter starts at \$5.00. A passenger must pay an initial cost of \$5.00 in addition to the per kilometre charge.

(i) Describe the relation between C and d in words.

The cost of a taxi ride is \$5.00 plus \$1.50 per kilometre.

(j) How much would it cost to take a 100 km taxi ride?

$$C = 1.5d + 5$$
$$= 1.5(100) + 5$$
$$= 150 + 5$$
$$= 155$$

The cost would be \$155.00.

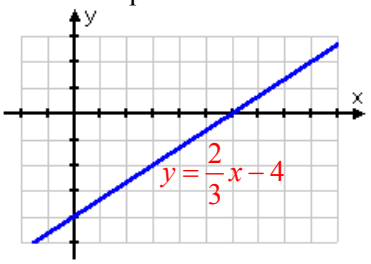
(k) Convert the equation that you obtained in (f) to standard form.

$$C = 1.5d + 5$$
$$\therefore 2C = 2(1.5d) + 2(5)$$
$$\therefore 2C = 3d + 10$$
$$\therefore 2C - 2C = 3d + 10 - 2C$$
$$\therefore 0 = 3d - 2C + 10$$
$$\therefore 3d - 2C + 10 = 0$$

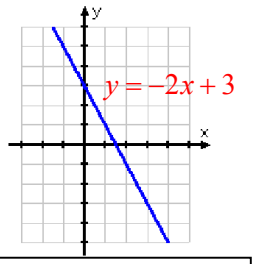
(l) Is there an easy way to determine the slope and y-intercept from the standard form of a linear relation?

Yes there is! See the next page for details.

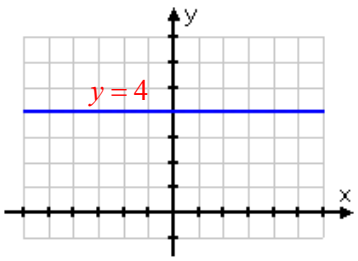
2. Write an equation for each of the following.



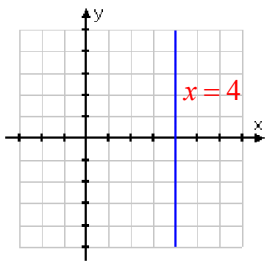
Positive Slope
Positive Rate of Change
As x increases, y increases.



Negative Slope
Negative Rate of Change
As x increases, y decreases.



Zero Slope
Zero Rate of Change
As x increases, y is constant.



Undefined Slope
Undefined Rate of Change
x is constant, y varies freely.

Detailed Answer to Question 1(l)

Begin with the ***standard form*** of a linear equation and perform operations to both sides until the slope-y-intercept form is obtained:

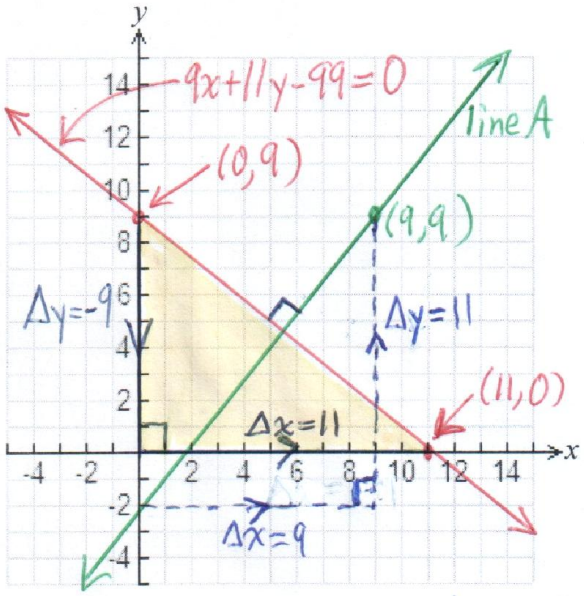
$$Ax + By + C = 0$$
$$\therefore Ax + By + C - Ax - C = 0 - Ax - C$$
$$\therefore By = -Ax - C$$
$$\therefore \frac{By}{B} = -\frac{A}{B}x - \frac{C}{B}$$
$$\therefore y = -\frac{A}{B}x - \frac{C}{B}$$

Comparing to the equation $y = mx + b$, we see that $m = -\frac{A}{B}$ and $b = -\frac{C}{B}$.

For example, for the linear equation $2x - 5y - 7 = 0$, $m = -\frac{A}{B} = -\left(\frac{2}{-5}\right) = \frac{2}{5}$ and $b = -\frac{C}{B} = -\left(\frac{-7}{-5}\right) = -\frac{7}{5}$

Important Problem Set

1. Line A passes through the point $(9,9)$ and is ***perpendicular*** to the line with equation $9x+11y-99=0$.



(a) Using the provided set of axes, sketch the graph of $9x+11y-99=0$. The fastest approach is to use the intercepts method.

(b) On the same set of axes, carefully sketch the graph of line A. Make sure that line A passes through $(9,9)$ and that it is perpendicular to $9x+11y-99=0$.

(c) Use your sketch of line A to estimate its slope and y-intercept.

$m \doteq \frac{11}{9}$ $\quad b \doteq -2$

(d) Determine the *exact* slope of the line $9x+11y-99=0$. Show your work and state a conclusion.

There are many ways to answer this question. One way is to write the equation in slope-intercept form.

$9x+11y-99=0$

$\therefore 9x+11y-99-9x+99=0-9x+99$

$\therefore 11y=-9x+99$

$\therefore \frac{11y}{11}=\frac{-9x}{11}+\frac{99}{11}$

$\therefore y=-\frac{9}{11}x+9$

Therefore, the slope of $9x+11y-99=0$ is $-\frac{9}{11}$.

x-intercept $\rightarrow y=0$

$\therefore 9x+11(0)-99=0$

$\therefore 9x-99=0$

$\therefore 9x=99$

$\therefore x=11$

y-intercept $\rightarrow x=0$

$\therefore 9(0)+11y-99=0$

$\therefore 11y-99=0$

$\therefore 11y=99$

$\therefore y=9$

(e) Use your answer from (d) to determine the slope of line A.

For line A, $m=\frac{11}{9}$ because line A is perpendicular to $9x+11y-99=0$ so their slopes are negative reciprocal

(f) Use your answer from (e) and the fact that line A passes through the point $(9,9)$ to determine the equation of line A.

line A: $m=\frac{11}{9}$, $b=?$, passes through $(9,9)$

Therefore, the equation of line A is of the form $y=\frac{11}{9}x+b$.

Since $(9,9)$ lies on line A,

$9=\frac{11}{9}(\frac{9}{1})+b$

$\therefore 9=11+b$

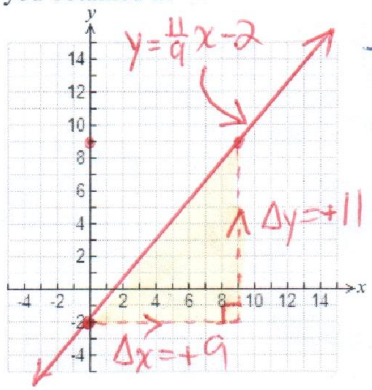
$\therefore 9-11=11+b-11$

$\therefore -2=b$

$\therefore$ the equation of line A is

$y=\frac{11}{9}x-2$

(g) Now check your answer to (f) by using the equation that you obtained in (f) to sketch the graph of line A. Does it agree with the graph that you obtained in



The graphs agree!

(Graph plotted using y-intercept and slope)

(h) Now summarize your results.

Estimates from (c)	*Actual Values from (f)*	*Conclusion(s): Is your equation for line A correct?*
$m \doteq \frac{11}{9}$	$m=\frac{11}{9}$	The exact values obtained are identical to the estimates. My answer is probably right.
$b \doteq -2$	$b=-2$	

2. Line A passes through the point $(9,9)$ and is ***parallel*** to the line with equation $9x+11y-99=0$.
Determine an equation of line A.

slope of $9x+11y-99=0$ is $-\frac{9}{11}$ (from #1)
$\therefore$ slope of line A is also $-\frac{9}{11}$ (parallel lines)
$\therefore$ equation is of form $y=-\frac{9}{11}x+b$
$\therefore$ line A passes through $(9,9)$

$9=-\frac{9}{11}(9)+b$
$\therefore 9=-\frac{81}{11}+b$
$\therefore 9+\frac{81}{11}=b$
$\therefore b=\frac{81}{11}+\frac{99}{11}=\frac{180}{11}$

$\therefore$ the equation of line A is $y=-\frac{9}{11}x+\frac{180}{11}$

3. The equation $n-E+15=0$ describes the amount earned per hour in a certain factory. In this equation, E represents the amount earned per hour in dollars and n represents the number of years of experience.

Calculate the hourly earnings of a beginning factory worker as well as one with five years of experience.

Beginner: $n=0$
$\therefore 0-E+15=0$
$\therefore -E+15=0$
$\therefore -E+15+E=0+E$
$\therefore E=15$
A beginner gets paid \$15.00/h

Five Years of Experience: $n=5$
$\therefore 5-E+15=0$
$\therefore -E+20=0$
$\therefore -E+20+E=0+E$
$\therefore 20=E$
$\therefore$ a worker with 5 years of experience earns \$20/h.

4. The equation $9C-5F+160=0$ describes the relationship between temperature, C, in degrees Celsius and temperature, F, in degrees Fahrenheit.

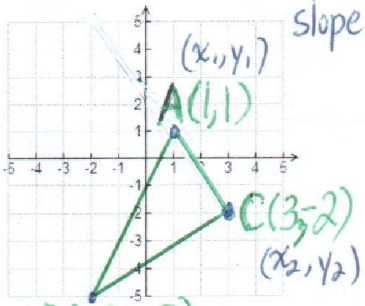
(a) Express the equation in the form $C=mF+b$.

$9C-5F+160=0$
$\therefore 9C-5F+160+5F-160=0+5F-160$
$\therefore 9C=5F-160$
$\therefore \frac{9C}{9}=\frac{5F}{9}-\frac{160}{9}$
$\therefore C=\frac{5}{9}F-\frac{160}{9}$

(b) Explain the ***meaning*** of the slope and the vertical intercept.

slope: $\frac{5}{9}$ degrees Celsius per degree Fahrenheit ($1°F=\frac{5}{9}°C$)
vertical intercept: $0°F=-\frac{160}{9}°C\doteq -17.8°C$

5. Plot the points $A(1,1)$, $B(-2,-5)$ and $C(3,-2)$ to form ΔABC. Is ΔABC a right triangle? Justify your answer using ***mathematical reasoning***.



slope of $AC=\frac{-2-1}{3-1}=-\frac{3}{2}$

slope of $BC=\frac{-2-(-5)}{3-(-2)}=\frac{-2+5}{3+2}=\frac{3}{5}$

Since the slopes are not negative reciprocals of each other $AC \not\perp BC$, ΔABC is not a right triangle

6. Given that A and k are one-digit numbers, determine the numbers of pairs of values for which the lines $Ax-3y+15=0$ and $y=kx+7$ are

(a) parallel
(b) perpendicular
(c) coincident (the same line)

l_1: $Ax-3y+15=0$
l_2: $y=kx+7$

The slope of l_2 is k.

The slope of l_1 can be determined by converting to slope-y-intercept form.

$Ax-3y+15=0$
$\therefore -3y=-Ax-15$
$\therefore \frac{-3y}{-3}=\frac{-Ax}{-3}-\left(\frac{15}{-3}\right)$
$\therefore y=\frac{A}{3}x+5$

The slope of l_1 is $\frac{A}{3}$.

(a) parallel

$k=\frac{A}{3}$

A	k
3	1
6	2
9	3
-3	-1
-6	-2
-9	-3
0	0

(b) perpendicular

$k=-\frac{3}{A}$

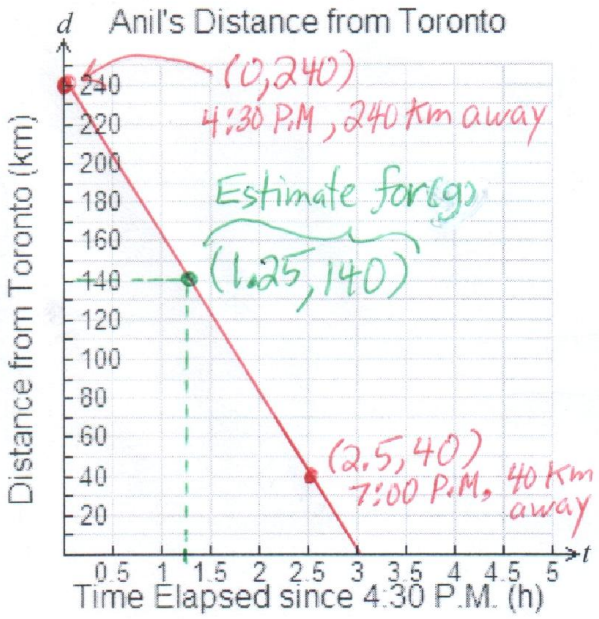
A	k
1	-3
3	-1
-1	3
-3	1

(c) coincident

The lines cannot be coincident because their y-intercepts are not equal.

7. Anil is driving to his home in Toronto at a ***constant speed***. At 4:30 P.M., he spots a sign indicating that Toronto is 240 km away. At 7:00 P.M., he notices another sign indicating that Toronto is 40 km away.

(a) Plot a graph showing Anil's distance from Toronto versus time elapsed since 4:30 P.M.



(b) Do you expect the relation between distance from Toronto and time to be linear? Explain.

It is linear because the speed is constant. (Speed is the rate of change of distance over time)

(c) Let t represent time in hours and d represent distance from Toronto in km. Write an equation relating d to t. Show all work!

$$\text{slope} = m = \frac{\Delta d}{\Delta t} = \frac{d_2 - d_1}{t_2 - t_1} = \frac{40 - 240}{2.5 - 0}$$

$$= \frac{-200}{2.5} = -80$$

$y\text{-intercept} = b = 240$

$\therefore d = -80t + 240$

(d) How fast is Anil travelling? Explain.

The speed is 80 km/h (slope). The slope is negative because the distance from Toronto decreases with time.

(e) Explain the ***meaning*** of the y-intercept of the graph in (a).

The y-intercept give the distance from Toronto at $t=0$ (4:30 P.M.)

(f) Explain the ***meaning*** of the x-intercept of the graph in (a).

The x-intercept is the time at which Anil arrived in Toronto.

(g) At 5:45 P.M., ***how far*** from Toronto was Anil? Determine this by using both the graph and the equation. Make sure that your answers agree!

Estimate from Graph: 140 km

$t = 1.25$

Exact Distance using Equation (Show all Work)

At 5:45 P.M., $t = 1.25$

$\therefore d = -80(1.25) + 240$

$= -100 + 240$

$= 140$

The answers agree!

At 5:45 P.M., Anil was 140 km from Toronto.

(h) ***At what time*** did Anil arrive in Toronto? Determine this by using both the graph and the equation. Make sure that your answers agree!

Estimate from Graph: 3 h after 4:30 (7:3

Exact Time using Equation (Show all Work)

When Anil arrives in Toronto, he is 0 km from Toronto. Therefore, $d = 0$

$\therefore 0 = -80t + 240$

$\therefore 0 - 240 = -80t + 240 - 240$

$\therefore -240 = -80t$

$\therefore \frac{-240}{-80} = \frac{-80}{-80}$

$\therefore 3 = t$

The answers agree!

(i) Anil was rushing home to Toronto because he did not want to miss watching the Maple Leafs lose yet another game. If the opening faceoff was to take place at 7:45 P.M., did Anil make it home in time? Explain.

Anil arrived home 3h after 4:30 P.M., that is, at 7:30 P.M. He made it home in plenty of time to watch the Leafs lose.

Solving System of Two Linear Equations – Applications

Example 1:

V – represents volume of gas in litres
C – represents cost of gas in $

All Natural is a gas station that charges customers $1.00 per litre of gas. It costs All Natural $0.50 per litre plus a flat fee of $20 to obtain gas from their oil supplier.

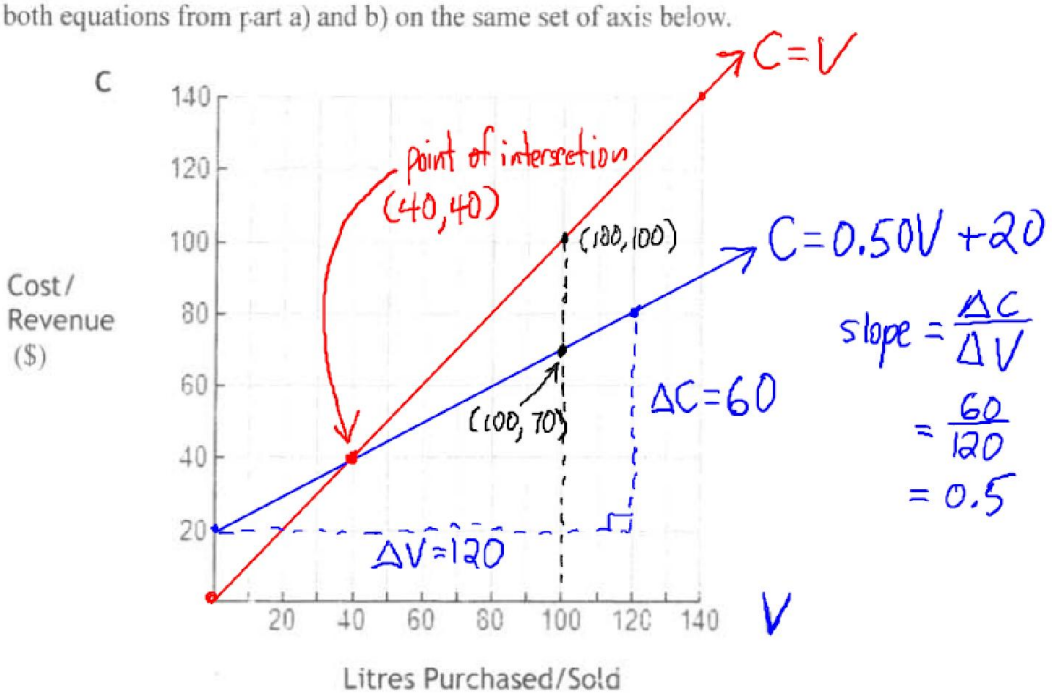
a) Write an equation that represents the cost for All Natural to obtain gas from the supplier.

$C = 0.50V + 20$

b) Write an equation that represents the revenue All Natural earns from selling gas to customers.

$C = 1.00V$ (or just $C = V$)

c) Graph both equations from part a) and b) on the same set of axis below.



d) State the point of intersection of the two lines.

(40, 40)

e) What does the point of intersection mean in this situation?

This is the "break even" point. If 40 litres of gasoline are sold, "All Natural" makes just enough money to cover the cost of the gasoline.

f) By looking at the graph, determine if All Natural will make money or lose money if 100 litres of gas is sold. What is the profit/loss?

From the graphs we can see that it costs "All Natural" $70 to purchase 100 L of gasoline from the supplier but they make $100 from selling 100 L of gasoline.

Therefore, the profit is $100.0 - $70.00 = $30.00

g) By using the equations in part a) and b), determine if All Natural will make money or lose money if 60 litres of gas is sold.

Pay to supplier: $C = 0.50(60) + 20 = 30 + 20 = 50$

Make from selling 60 L: $C = V = 60$

∴ All Natural will make a profit of $60 - $50 = $10

h) Will All Natural make money or lose money if 20 litres of gas is sold? You may use the graph or the equations to find your answer.

Pay to Supplier: $C = 0.50(20) + 20 = 10 + 20 = 30$

Make from selling 20 L: $C = V = 20$ ∴ All Natural will lose $10

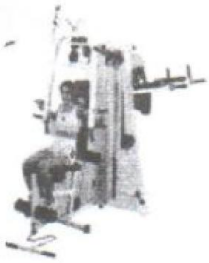
i) **Conclusions:**

If less than 40 litres of gas is sold, All Natural will have a loss.

If exactly 40 litres of gas is sold, All Natural will break even.

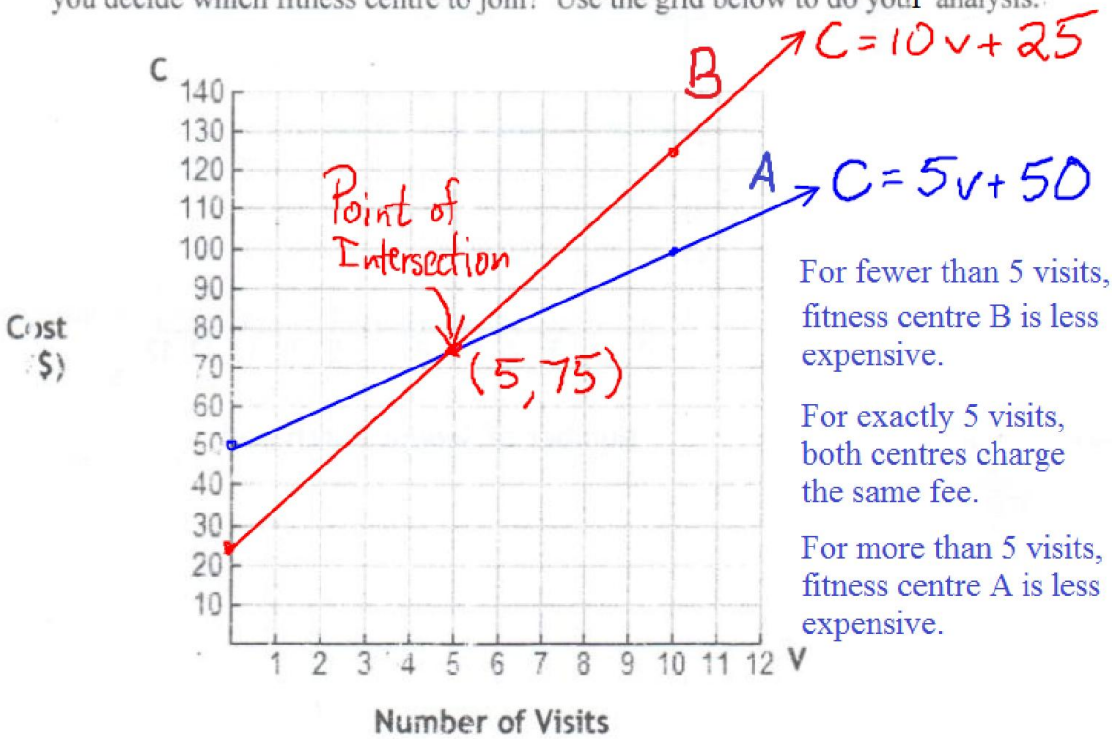
If more than 40 litres of gas is sold, All Natural will have a profit.

Example 2:



Let C represent the cost, v represent the # of visits

Fitness Centre A charges an annual membership fee of $50 plus $5 for each visit. **Fitness Centre B** charges an annual membership fee of $25 plus $10 for each visit. How would you decide which fitness centre to join? Use the grid below to do your analysis.



For fewer than 5 visits, fitness centre B is less expensive.

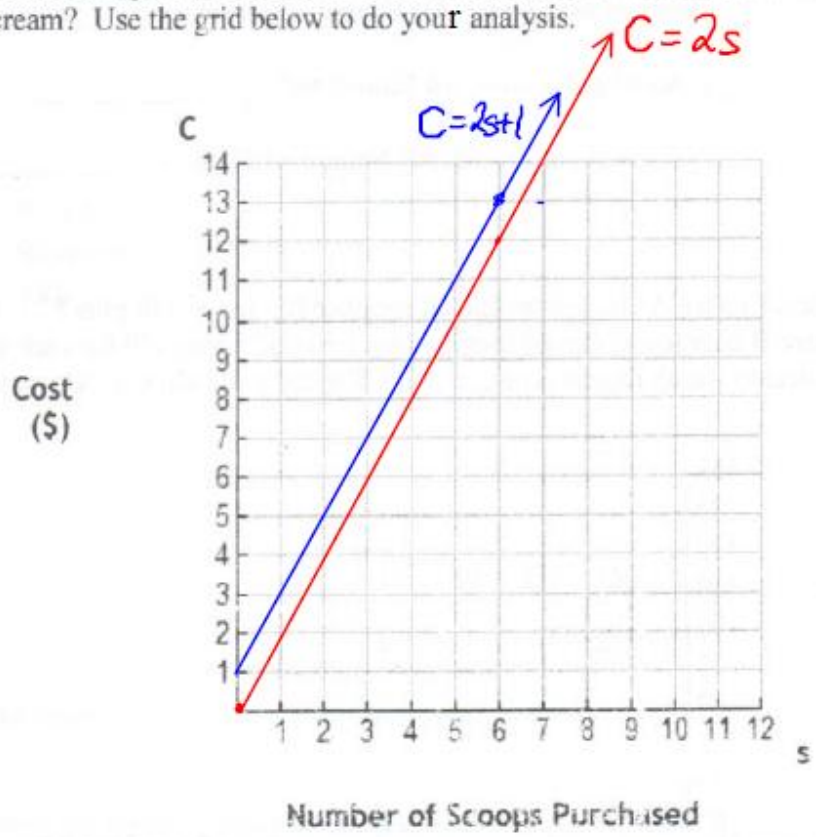
For exactly 5 visits, both centres charge the same fee.

For more than 5 visits, fitness centre A is less expensive.

Let C represent the cost, s represent # scoops

Example 3

Pure Dairy charges \$2 for a scoop of ice cream (cone included). **Cones 'R Us** charges \$2 for a scoop of ice cream plus an additional \$1 for the cone. How would you decide where to buy your ice cream? Use the grid below to do your analysis.

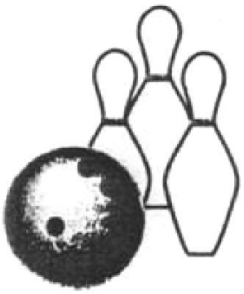


Since the lines have the same slope, they must be parallel. Thus, they never intersect. The line $C=2s+1$ is always above the line $C=2s$, which means that the Cones 'R Us price is always higher for a given number of scoops.

Unless the Cones 'R Us ice cream happens to taste much better than Pure Dairy ice cream, my choice would be Pure Dairy!

Solutions – Task 1: Bowling

Task 1: Bowling!



A group of 4 friends is going bowling at **Bowling Bonanza.**
Bowling Bonanza charges

- $2.50 for each player to rent shoes
  plus
- $20/h for a group of 4 to bowl.

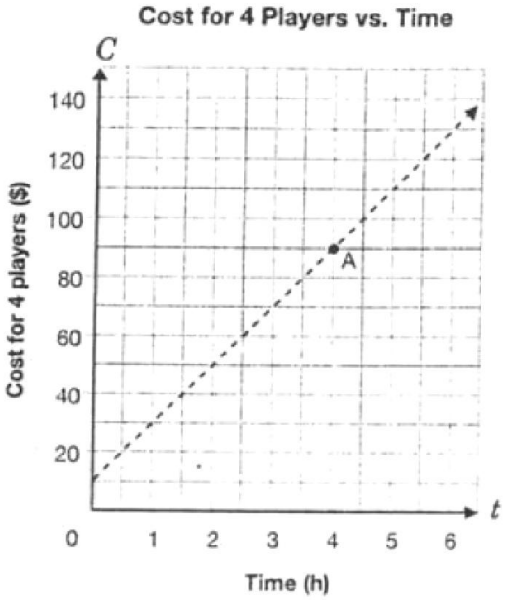
a) The graph below represents the relationship between cost, C, in dollars, and time, t, in hours, for 4 players to bowl.

i) **Write the coordinates** of point A.

(4, 90)

ii) **Explain** what the coordinates of point A tell you about the cost of bowling.

It costs $90.00 for four players to bowl for four hours.



b) **Explain** how this graph would change if the cost for renting the shoes increased.

The slope would stay the same because the hourly rate would not change. The y-intercept would be higher, however, producing a parallel line above the given line.

Hint:
Refer to slope and y-intercept.

c) **Circle** the equation that represents the graph in question a).

$C = 20t + 10$ $\qquad$ $C = 20t^2 + 10$ $\qquad$ $C = \frac{20}{t} + 10$

Give reasons for your answer.

- It's the only linear equation.
- The slope is 20 and the y-intercept is 10, which matches the graph.

d) This group of friends wants to spend $80.
How many hours can they bowl at **Bowling Bonanza?**
Give reasons for your answer or show your work.

$C = 20t + 10$

Solve for t

$C - 10 = 20t + 10 - 10$

$\therefore C - 10 = 20t$

$\therefore \frac{C-10}{20} = \frac{20t}{20}$

$\therefore \frac{C-10}{20} = t$

$\therefore t = \frac{C-10}{20}$

If $C = 80$,

then $t = \frac{80-10}{20}$

$= \frac{70}{20}$

$= 3.5$

For $80, the friends can play for 3.5 hours

Note: This problem can also be solved graphically (see graph previous page)

e) William **and** his 3 friends are going bowling.
He finds an advertisement in the newspaper for a new bowling alley, **Super Bowl.**
William and his friends will play 6 games in 3 hours.

Determine whether William and his friends should go bowling at **Bowling Bonanza** or **Super Bowl.** Use the information given in the advertisement and in the hint box.

Give reasons for your answer.

Super Bowl

Cost = 6(4)($3.00)

= $72.00

Bowling Bonanza

Cost = 4($2.50) + $20(3)

= $10.00 + $60.00

= $70.00

Bowling Bonanza is a slightly better deal.

Hint:
Bowling Bonanza charges
- $2.50 for each player to rent shoes and
- $20/h for a group of 4 to bowl.

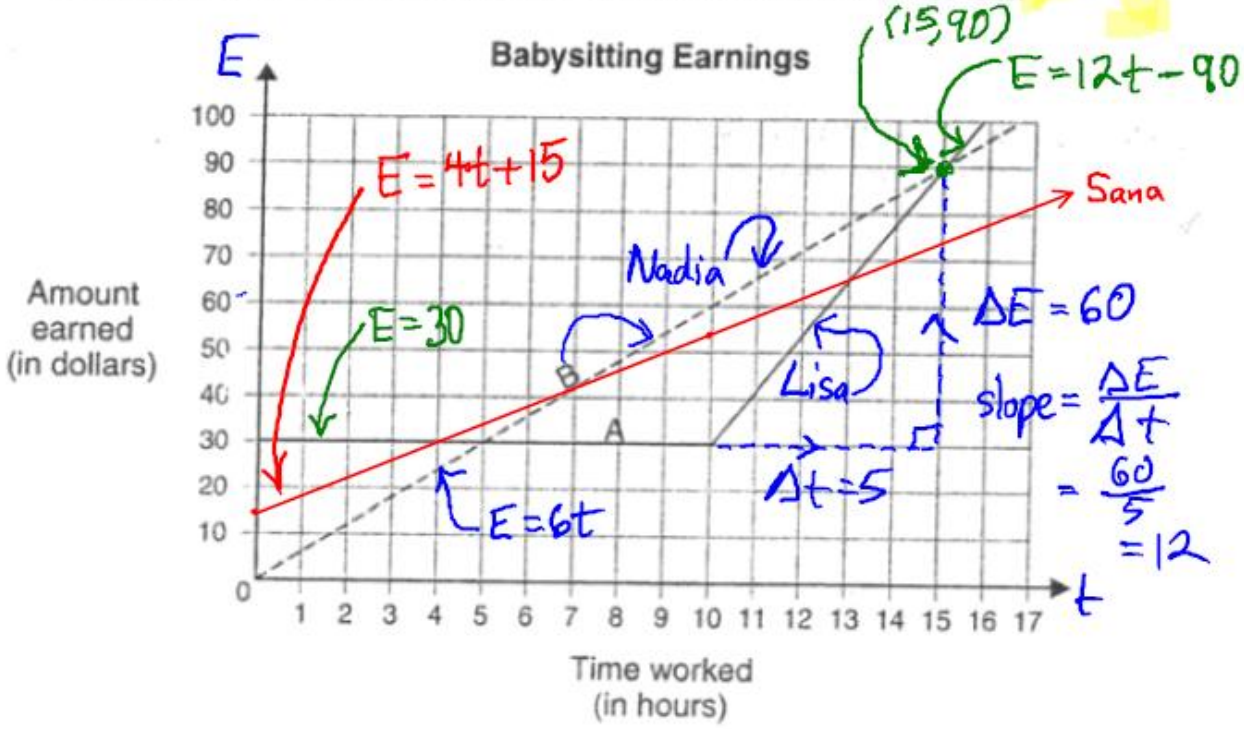
Task 2: Babysitters' Club

$E = 12t + b$
$90 = 12(15) + b$
$90 = 180 + b$
$\therefore b = 90 - 180$
$\therefore b = -90$

Nadia and Lisa are comparing their **weekly earnings** from babysitting. The following graph shows their earnings compared to the number of hours they worked in the week.

a) Lisa says:
"If we both work **less than 5 hours** or **more than 15 hours**, I earn more than you do."

Label Lisa's line with her name. Write Nadia's name on the other line.



b) Describe what the graph shows about how each girl is paid for her week of work. Include specific mathematical details about hourly rates of pay.

Lisa gets paid a flat fee of $30.00 for up to ten hours of work. For more than ten hours, she is paid an additional amount of $12/h (see graph above).

Nadia is simply paid at a rate of

$$\frac{\$90.00}{15\text{ h}} = \$6.00/\text{h}$$

c) Sana also offers babysitting in the home. She lives on the edge of town and travels by bus to the home where she babysits.

Sana charges a family a set fee of **$15.00 per week** to cover her bus pass **plus an additional $4.00 per hour**.

Draw the graph for Sana's earnings on the graph in question **a)**. Label your line. (See previous page for graph)

$E = 4t + 15$, E = amount earned in $

t = time worked in hours

d) Your neighbour needs a babysitter for 12 h this week.
How much would each of the three girls charge for this 12 h of babysitting?
Show your work or explain how you get each answer.

Sana: $E = 4(12) + 15 = 63$

Nadia: $E = 6(12) = 72$

Lisa: $E = 30 + 2(12) = 30 + 24 = 54$

e) Several neighbours have inquired about babysitters. Some require a lot of hours of babysitting per week while others require very few hours. They have asked you which of the babysitters charges the least. What would your answer be?
Explain your reasoning. Be specific about the time intervals.

Up to 5 hours ⟶ Nadia is the cheapest

Between 5 and 13 hours ⟶ Lisa is the cheapest

More than 13 hours ⟶ Sana is the cheapest

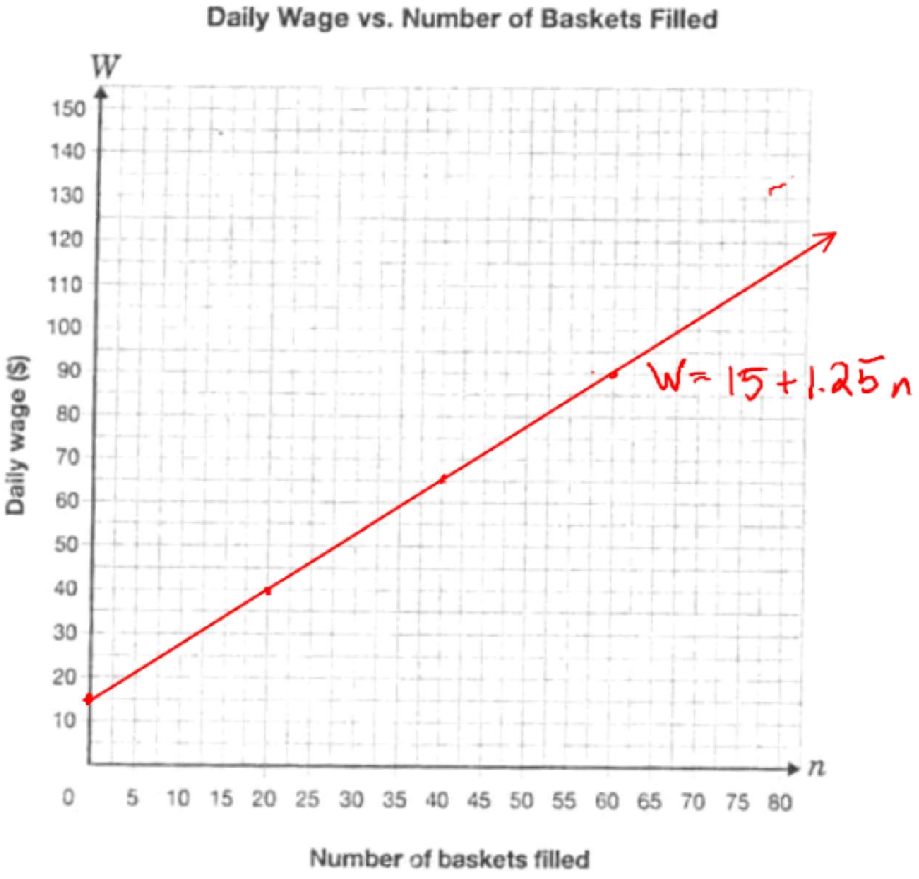
Open-Response

2. Berries for Picking

Sanya has a summer job picking berries at a farm. Each day, she is paid a base salary, plus an amount for each basket she fills with berries.

The equation $W = 15 + 1.25n$ represents the relationship between Sanya's **daily wage,** W, in dollars, and the **number of baskets** she fills, n.

a) Graph the relationship represented by the equation on the grid below.



n	W
0	15
20	40
40	65
60	90

b) Explain what the **slope** of the line means in relation to picking berries.

The slope is 1.25. It means that Sanya is paid $1.25 for every basket she fills.

c) Determine **the number of baskets** that Sanya must fill to have a daily wage of **$70.**
Show your work.

$W = 15 + 1.25n$

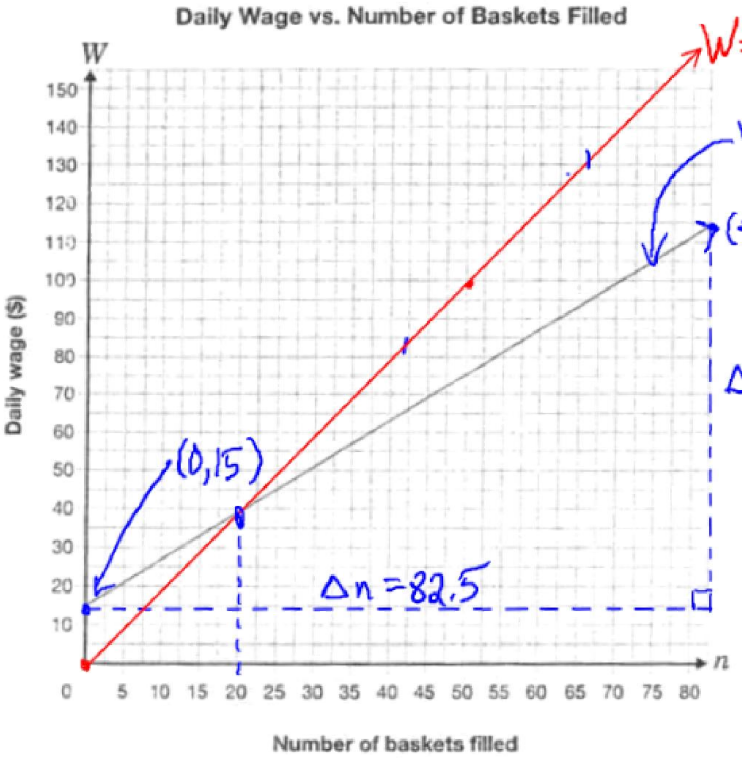
$\therefore W - 15 = 1.25n$

$\therefore n = \frac{W-15}{1.25}$

If $W = 70$ then $n = \frac{70-15}{1.25} = \frac{55}{1.25} = 44$

To earn $70.00, Sanya must fill 44 baskets.

d) Sanya's brother picks cucumbers at another farm. His payment structure is represented on the graph below.



slope $= \frac{\Delta W}{\Delta n}$

$= \frac{100}{82.5}$

$\doteq 1.21$

OR

Slope $= \frac{W_2 - W_1}{n_2 - n_1}$

$= \frac{115 - 100}{82.5 - 0}$

$\doteq 1.21$

He gets paid a flat fee of $15.00 (y-int.)

He is offered a **new** payment structure of $2.00 per basket but **no daily base salary.**

Should Sanya's brother accept this new payment structure?
Explain your answer.

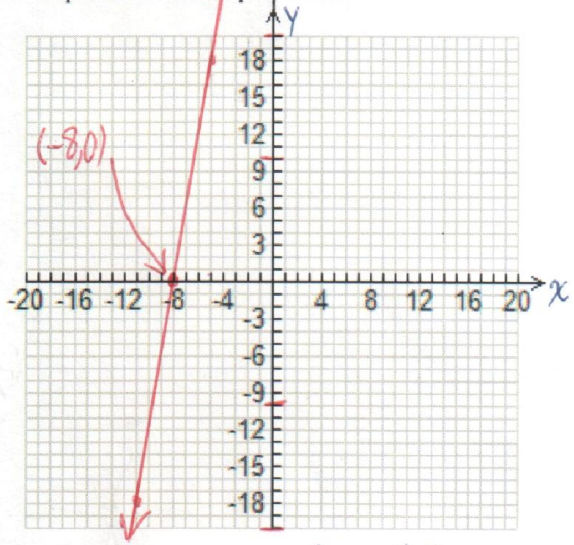
Sanya's brother should accept this payment structure only if he can pick more than 20 baskets per day. This can be seen from the graph above. If $n > 20$, the line $W = 2n$ lies above the line $W = 1.21n + 15$, meaning that the daily wage will be greater.

Solutions – Very Important Review of Linear Relations

Very Important Review of Linear Relations

1. Find an equation of with slope 6 and having x-intercept -8.

(a) Make a sketch and use it to ***estimate*** the equation of the required line.



Estimate of Equation: $y = 6x + 45$

(b) Use an algebraic method to find the ***exact*** equation of the required line.

slope $= m = 6$, $b = ?$ $y = 6x + b$

Since the line passes through $(-8, 0)$,

$0 = 6(-8) + b$

$\therefore 0 = -48 + b$

$\therefore 0 + 48 = -48 + b + 48$

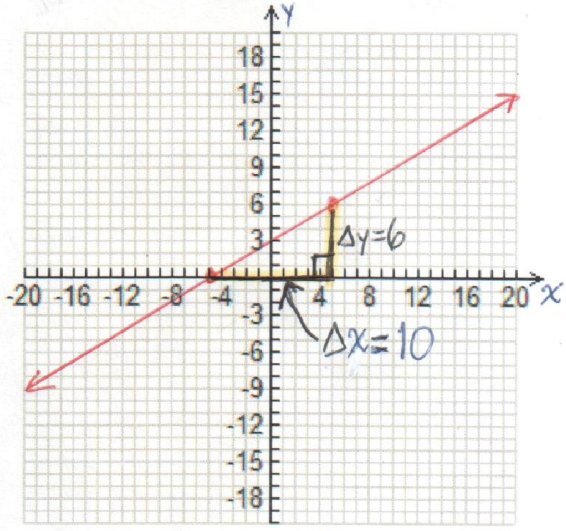
$\therefore 48 = b$

The slope, y-intercept equation is $\boxed{y = 6x + 48}$

2. Find an equation of the line that passes through $(-5, 0)$ and $(5, 6)$.

$x_1\ y_1 \quad x_2\ y_2$

(a) Make a sketch and use it to ***estimate*** the equation of the required line.



Estimate of Equation: $y = \frac{3}{5}x + 3$

$m = \frac{\Delta y}{\Delta x} = \frac{6}{10} = \frac{3}{5}$

(b) Use an algebraic method to find the ***exact*** equation of the required line.

$m = ?$, $b = ?$

$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6 - 0}{5 - (-5)} = \frac{6}{10} = \frac{3}{5}$

Therefore, the equation is of the form $y = \frac{3}{5}x + b$.

Since the line line passes through $(-5, 0)$,

$0 = \frac{3}{5}\left(\frac{-5}{1}\right) + b$

$\therefore 0 = -3 + b$

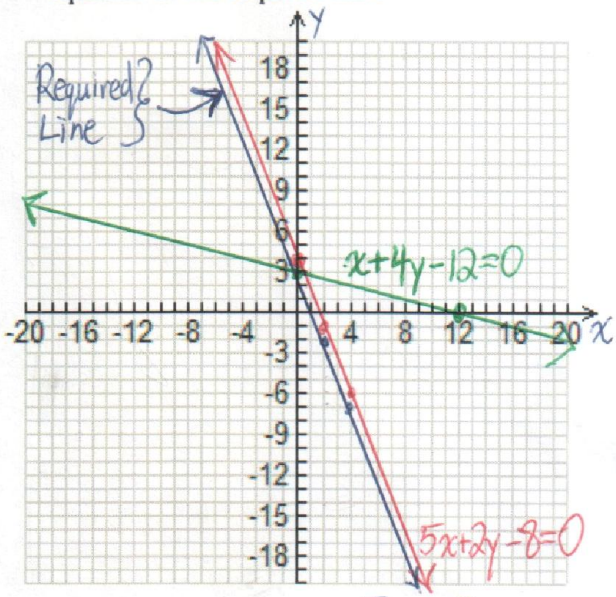
$\therefore b = 3$

The slope, y-intercept equation is $\boxed{y = \frac{3}{5}x + 3}$

same slope — same value of b

3. A line is parallel to $5x + 2y - 8 = 0$ and has the same y-intercept as $x + 4y - 12 = 0$. Find an equation of the line.

(a) Make a sketch and use it to ***estimate*** the equation of the required line.



Estimate of Equation: $y = -\frac{5}{2}x + 3$

(b) Use an algebraic method to find the ***exact*** equation of the required line. → $m = ?$, $b = ?$

(i) slope of $5x+2y-8=0$

$2y = -5x + 8$

$\therefore \frac{2y}{2} = \frac{-5}{2}x + \frac{8}{2}$

$\therefore y = -\frac{5}{2}x + 4$

$\therefore m = -\frac{5}{2}$ since the lines are parallel

(ii) y-intercept of $x+4y-12=0$

Let $x = 0$. Then,

$0 + 4y - 12 = 0$

$\therefore y = 3$

$\therefore b = 3$

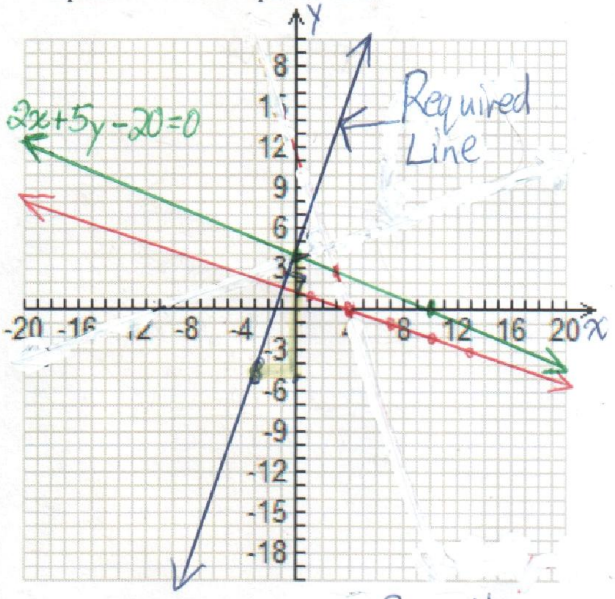
The slope, y-intercept equation of the required line is

$$y = -\frac{5}{2}x + 3$$

negative reciprocal slope — same value of b

4. A line is perpendicular to $x + 3y - 4 = 0$ and has the same y-intercept as $2x + 5y - 20 = 0$. Find an equation of the line.

(a) Make a sketch and use it to ***estimate*** the equation of the required line.



Estimate of Equation: $y = -3x + 4$

(b) Use an algebraic method to find the ***exact*** equation of the required line.

(i) slope of $x+3y-4=0$

$3y = -1x + 4$

$\therefore \frac{3y}{3} = \frac{-1}{3}x + \frac{4}{3}$

$\therefore y = -\frac{1}{3}x + \frac{4}{3}$

$\therefore m = \frac{3}{1}$ since the lines are perpendicular

(ii) y-intercept of $2x+5y-20=0$

Let $x = 0$. Then,

$2(0) + 5y - 20 = 0$

$\therefore y = 4$

$\therefore b = 4$

The slope, y-intercept equation of the required line is

$$y = -3x + 4$$