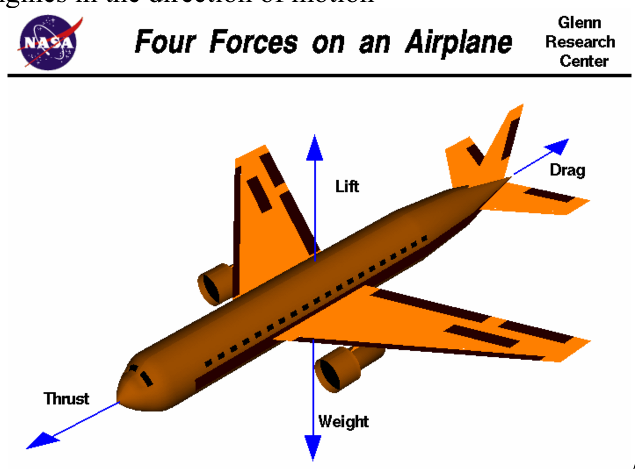
SOLUTIONS TO PROBLEMS ON PAGE 13 OF UNIT 2

1. Describe the forces acting on an aircraft flying at a constant speed at a constant altitude.

(a) **Weight**: force of gravity acting downward
(b) **Lift**: buoyant force of the atmosphere acting upward
(c) **Drag**: air resistance acting opposite the direction of motion
(d) **Thrust**: produced by the engines in the direction of motion



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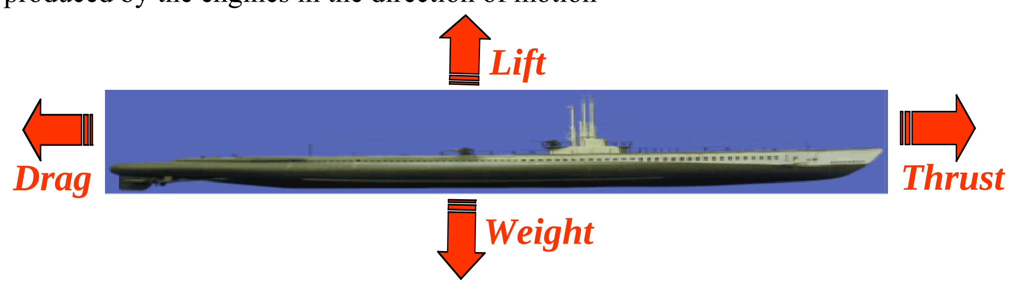
2. Describe the forces acting on a submarine cruising at a constant speed at a constant depth.

(a) **Weight**: force of gravity acting downward
(b) **Lift**: buoyant force of the water acting upward
(c) **Drag**: water resistance acting opposite the direction of motion
(d) **Thrust**: produced by the engines in the direction of motion



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3. Describe the forces acting on an automobile moving at a constant speed on a flat horizontal road.

(a) **Weight**: force of gravity acting downward
(b) **Normal Force**: force of the road counteracting the weight of the car (Newton's Third Law)
(c) **Drag**: air resistance and rolling friction acting opposite the direction of motion
(d) **Thrust**: produced by the engines in the direction of motion//

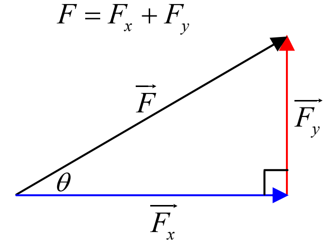
4. A force of 200 N is being applied to a rope to pull a toboggan along a frictionless, horizontal surface. If the rope forms an angle of 60° to the horizontal, find the horizontal and vertical components of the force (this is called ***resolving*** the vector). Which of the components does all the work, the vertical or the horizontal? What does this tell you about the angle at which the rope should be pulled? Should the angle be as close to 90° as possible or as close to 0° as possible?

Solution

Since a force of 200 N is applied, $\left|\vec{F}\right| = 200$.

When $\theta = 60°$, $\left|\vec{F_x}\right| = 200\cos 60° = 200(\tfrac{1}{2}) = 100$ and $\left|\vec{F_y}\right| = 200\sin 60° = 200(\tfrac{\sqrt{3}}{2}) = 100\sqrt{3}$.

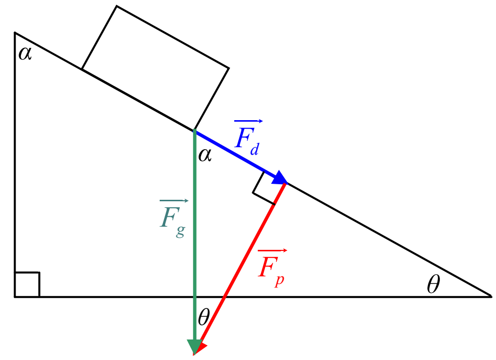
$\vec{F} = \vec{F_x} + \vec{F_y}$



Since the toboggan moves in the horizontal direction along the ground, the vertical component of the force does not do any work. Therefore, the most efficient way to pull the toboggan is to make the angle θ as small as possible. This has the effect of minimizing the magnitude of $\vec{F_y}$.//

5. A 100 kg object ***rests*** on a ramp inclined at a certain angle θ to the horizontal. Calculate the components of the force of gravity on the object that are parallel and perpendicular to the ramp. Express your answers in terms of θ.

Solution



Let $\overrightarrow{F_g}$ represent the force of gravity acting on the object. As always, this force is directed downward toward the centre of the Earth. Then ***resolve*** $\overrightarrow{F_g}$ into two perpendicular vectors, $\overrightarrow{F_d}$, representing the component of $\overrightarrow{F_g}$ acting ***down*** the ramp and $\overrightarrow{F_p}$, the component acting ***perpendicular*** to the ramp. Clearly, $\overrightarrow{F_g} = \overrightarrow{F_d} + \overrightarrow{F_p}$. Since the vector $\overrightarrow{F_g}$ is parallel to the end of the ramp, the angles marked "α" must be equal. This means that the right triangle formed by the three vectors is ***similar*** to the right triangle formed by the ramp (AA similarity). Finally, we can calculate the magnitudes of the components of $\overrightarrow{F_g}$.

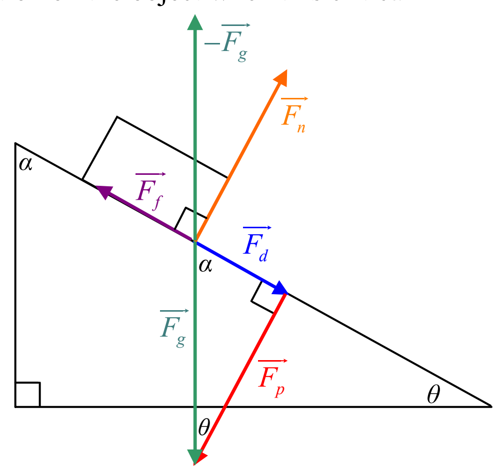
Since $\left|\overrightarrow{F_g}\right| = mg \doteq (100\text{ kg})(9.8\text{ m/s}^2) = 980\text{ kg m/s}^2 = 980\text{ N}$, ($g$ is the acceleration due to gravity)

$$\left|\overrightarrow{F_d}\right| \doteq 980\sin\theta \text{ and } \left|\overrightarrow{F_p}\right| \doteq 980\cos\theta.$$

More generally, if the mass of the object is m, then $\left|\overrightarrow{F_g}\right| = mg$, $\left|\overrightarrow{F_d}\right| = mg\sin\theta$ and $\left|\overrightarrow{F_p}\right| = mg\cos\theta$.//

6. Suppose now that the angle θ (in question 5) is increased slowly until a critical angle θ_c is reached and the object begins to slide down the ramp. What can you conclude about the force of friction on the object when this critical angle is reached?

Solution



When $\theta < \theta_c$, the object on the ramp remains stationary because the forces acting on it are ***balanced***. The force $\overrightarrow{F_d}$ acting down the ramp is exactly balanced by the force $\overrightarrow{F_f}$ (the force of ***friction***) acting up the ramp. Similarly, the force $\overrightarrow{F_p}$ acting perpendicular to and down from the ramp is exactly balanced by the force $\overrightarrow{F_n}$ (the ***normal*** force) acting perpendicular to and up from the ramp.

Mathematically, we can write this as shown below.

When $\theta < \theta_c$,

$\overrightarrow{F_d} + \overrightarrow{F_f} = \vec{0}$, $\overrightarrow{F_p} + \overrightarrow{F_n} = \vec{0}$, and $\overrightarrow{F_d} + \overrightarrow{F_f} + \overrightarrow{F_p} + \overrightarrow{F_n} = \vec{0}$

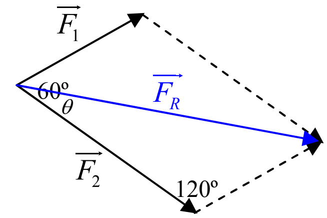
As θ increases, $\left|\overrightarrow{F_d}\right|$ increases. As long as θ remains small enough, $\left|\overrightarrow{F_f}\right|$ will increase exactly at the same rate as and remain equal to $\left|\overrightarrow{F_d}\right|$, ensuring that the object remains stationary. Once the critical angle θ_c is reached, however, $\left|\overrightarrow{F_d}\right|$, the magnitude of the component of $\overrightarrow{F_g}$ acting down the ramp, will overcome the force of friction and the object will slide down the ramp.

In a similar manner, if the object placed on the ramp is heavy enough, the ramp will not be able to exert a normal force on the object strong enough to hold it up. This would cause the ramp to collapse.//

7. Two draft horses pull a load. The chains between the horses and the load are at an angle of 60° to each other. One horse pulls with a force of 230 N and the other pulls with a force of 340 N. What is the resultant force on the load? What is the equilibrant force on the load? (State both magnitude and direction.)

Solution

As shown in the diagram, the parallelogram law is used to obtain the resultant force $\vec{F_R}$ ($\vec{F_R} = \vec{F_1} + \vec{F_2}$). We are given that $|\vec{F_1}| = 230$ and $|\vec{F_2}| = 340$. In addition, since the angle between $\vec{F_1}$ and $\vec{F_2}$ is 60º, the PLT theorem informs us that the other base angle of the parallelogram must be 120º. Then, by the law of cosines,



$$\begin{aligned}
|\vec{F_R}|^2 &= |\vec{F_1}|^2 + |\vec{F_2}|^2 - 2|\vec{F_1}||\vec{F_2}|\cos 120° \\
&= 230^2 + 340^2 - 2(230)(340)(-\tfrac{1}{2}) \\
&= 246700
\end{aligned}$$

Thus, $|\vec{F_R}| = \sqrt{246700} = 10\sqrt{2467} \doteq 496.7$ N. Now let θ represent the angle between $\vec{F_2}$ and $\vec{F_R}$. By applying the law of sines, we obtain

$$\begin{aligned}
\frac{\sin\theta}{|\vec{F_1}|} &= \frac{\sin 120°}{|\vec{F_R}|} \\
\therefore \sin\theta &= \frac{|\vec{F_1}|\sin 120°}{|\vec{F_R}|} \\
&= \frac{230(\frac{\sqrt{3}}{2})}{10\sqrt{2467}} \\
\therefore \theta &= \sin^{-1}\left(\frac{230(\frac{\sqrt{3}}{2})}{10\sqrt{2467}}\right) \\
&\doteq 23.6°
\end{aligned}$$

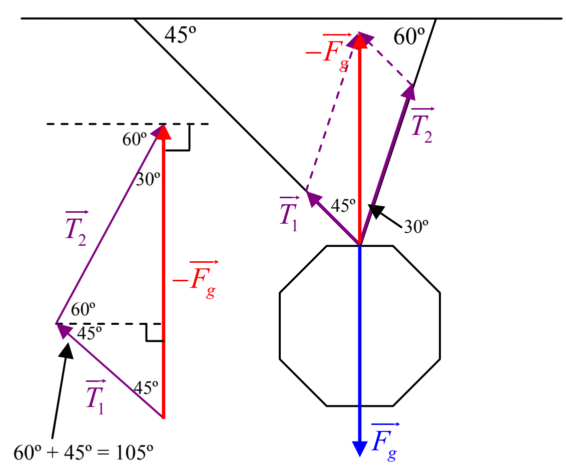
Therefore, the magnitude of the resultant force is about 496.7 N and the angle between $\vec{F_2}$ and $\vec{F_R}$ is about 23.6º. (Alternatively, we could state that the angle between $\vec{F_1}$ and $\vec{F_R}$ is about 37.4º.)//

8. A traffic sign with a mass of 5 kg is suspended above a street by two cables. One cable forms an angle of 45° to the street and the other forms an angle of 60°. Find the tension of each wire.

Solution

Let $\vec{T_1}$ and $\vec{T_2}$ represent the forces of tension in the two cables. Since the traffic sign is stationary, $\vec{T_1} + \vec{T_2} + \vec{F_g} = \vec{0}$. That is, the ***resultant*** force on the traffic sign is $\vec{0}$. (Alternatively, we can write that $-\vec{F_g} = \vec{T_1} + \vec{T_2}$. The sum of the forces of tension exactly counterbalance the force of gravity.)

Using ASTT and PLT, we find that the angle between $\vec{T_1}$ and $-\vec{F_g}$ is 45º and the angle between $-\vec{F_g}$ and $\vec{T_2}$ is 30º.



Now, $|-\vec{F_g}| = |\vec{F_g}| = mg \doteq (5\text{ kg})(9.8\text{ m/s}^2) = 49\text{ kg m/s}^2 = 49\text{ N}$.

Therefore, by applying the law of sines we obtain

$$\frac{|\vec{T_1}|}{\sin 30°} = \frac{|\vec{F_g}|}{\sin 105°} \doteq \frac{49}{\sin 105°} \quad \Rightarrow \quad |\vec{T_1}| = \frac{49\sin 30°}{\sin 105°} \doteq 25.4\text{ N and}$$

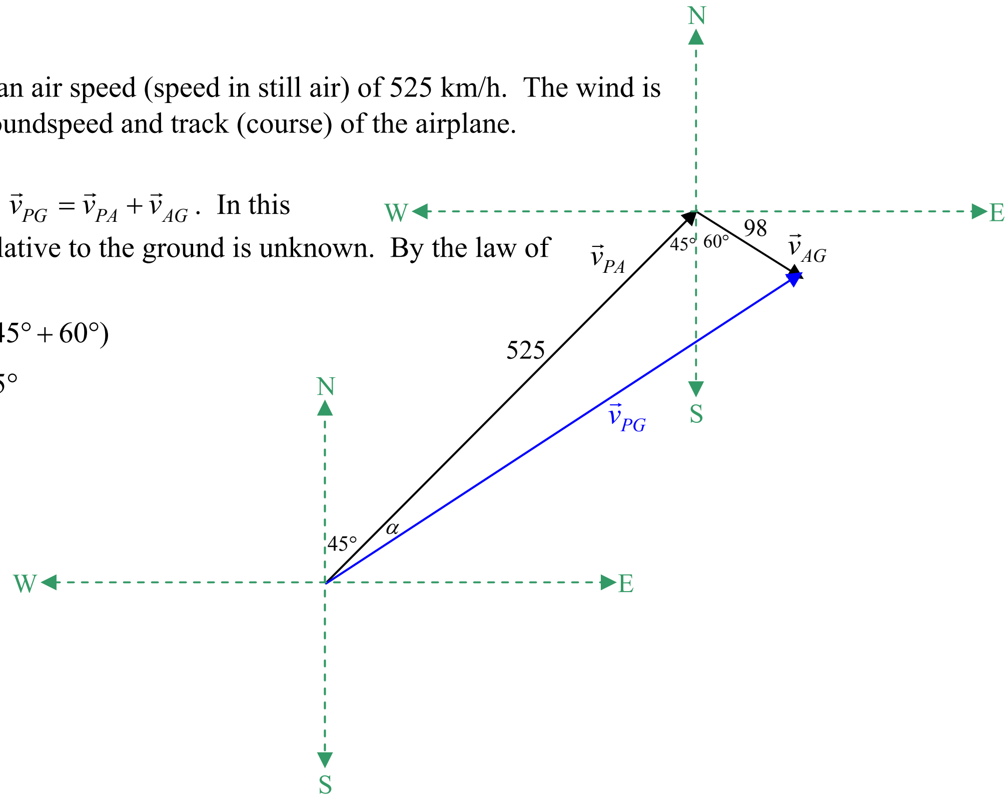
$$\frac{|\vec{T_2}|}{\sin 45°} = \frac{|\vec{F_g}|}{\sin 105°} \doteq \frac{49}{\sin 105°} \quad \Rightarrow \quad |\vec{T_2}| = \frac{49\sin 45°}{\sin 105°} \doteq 35.9\text{ N .//}$$

Therefore, the tension in the longer cable is about 25 N and the tension in the shorter cable is about 36 N.

9. An airplane is steering at N45°E with an air speed (speed in still air) of 525 km/h. The wind is ***from*** N60°W at 98 km/h. Find the groundspeed and track (course) of the airplane.

Solution

First we recall that $\vec{v}_{PA} = \vec{v}_{PG} - \vec{v}_{AG}$ or $\vec{v}_{PG} = \vec{v}_{PA} + \vec{v}_{AG}$. In this problem, the velocity of the aircraft relative to the ground is unknown. By the law of cosines,



$$\begin{aligned}|\vec{v}_{PG}|^2 &= |\vec{v}_{PA}|^2 + |\vec{v}_{AG}|^2 - 2|\vec{v}_{PA}||\vec{v}_{AG}|\cos(45° + 60°)\\ &= 525^2 + 98^2 - 2(525)(98)\cos 105°\\ &\doteq 311861.4797\end{aligned}$$

$$|\vec{v}_{PG}| \doteq 558.4 \text{ km/h}$$

By the law of sines,

$$\frac{\sin\alpha}{|\vec{v}_{AG}|} = \frac{\sin 105°}{|\vec{v}_{PG}|}$$

$$\begin{aligned}\therefore \sin\alpha &= \frac{|\vec{v}_{AG}|\sin 105°}{|\vec{v}_{PG}|}\\ &= \frac{98\sin 105°}{\sqrt{525^2 + 98^2 - 2(525)(98)\cos 105°}}\end{aligned}$$

$$\begin{aligned}\therefore \alpha &= \sin^{-1}\left(\frac{98\sin 105°}{\sqrt{525^2 + 98^2 - 2(525)(98)\cos 105°}}\right)\\ &\doteq 9.8°\end{aligned}$$

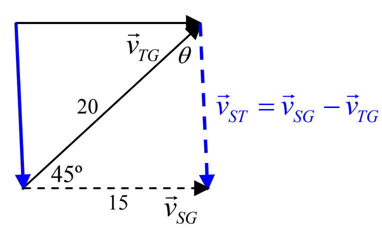
Therefore, the velocity of the aircraft is about 558.4 km/h, E35.2ºN.

10. A ship is steering east at 15 knots (nautical miles per hour). A tugboat 2 nautical miles (M) to the south is steering N45°E at 20 knots. Find the velocity of the ship relative to the tug. Will the ship pass in front of or behind the tug?

Solution

In this question we need to ***assume*** that the given velocities are measured relative to the same frame of reference, which for the sake of convenience we shall choose to be the ground (i.e. the shore). Let $\vec{v}_{ST}$ represent the velocity of the ship relative to the tug, $\vec{v}_{SG}$ represent the velocity of the ship relative to the ground and $\vec{v}_{TG}$ represent the velocity of the ship relative to the tug. Then, $\vec{v}_{ST} = \vec{v}_{SG} - \vec{v}_{TG}$ or equivalently, $\vec{v}_{SG} = \vec{v}_{ST} + \vec{v}_{TG}$.

Once again by the law of cosines,



$$\begin{aligned}|\vec{v}_{ST}|^2 &= |\vec{v}_{TG}|^2 + |\vec{v}_{SG}|^2 - 2|\vec{v}_{TG}||\vec{v}_{SG}|\cos 45°\\ &= 20^2 + 15^2 - 2(20)(15)(\tfrac{1}{\sqrt{2}})\end{aligned}$$

$$\begin{aligned}|\vec{v}_{ST}| &= \sqrt{20^2 + 15^2 - 2(20)(15)(\tfrac{1}{\sqrt{2}})}\\ &\doteq 14.17\end{aligned}$$

By the law of sines,

$$\frac{\sin\theta}{|\vec{v}_{SG}|} = \frac{\sin 45°}{|\vec{v}_{ST}|}$$

$$\begin{aligned}\therefore \sin\theta &= \frac{|\vec{v}_{SG}|\sin 45°}{|\vec{v}_{ST}|}\\ &= \frac{15\sin 45°}{\sqrt{20^2 + 15^2 - 2(20)(15)(\tfrac{1}{\sqrt{2}})}}\end{aligned}$$

$$\begin{aligned}\therefore \theta &= \sin^{-1}\left(\frac{15\sin 45°}{\sqrt{20^2 + 15^2 - 2(20)(15)(\tfrac{1}{\sqrt{2}})}}\right)\\ &\doteq 48.5°\end{aligned}$$

Therefore, the velocity of the ship relative to the tug is about 14.17 knots, S3.5ºE.

Will the ship pass in front of or behind the tug?

This part is to be done as a homework exercise.