

Grade 12 Geometry and Discrete Mathematics
Major Test – Unit 0 (Review of Essential Mathematical Concepts)

*Mark allotment
corrections*

Mr. N. Nolfi

Victim:

Mr. Solutions

*Once again, your work
is superb Mr. N.!!*

KU	APP	TIPS	COM
7/7	11/11	14/14	16/16

1. State whether each of the following is true or false. To receive full credit, you must prove the statements that are true and provide a counterexample or explanation for the statements that are false. (Remember that the best counterexamples are usually very simple!) (5 TIPS, 5 COM)

Statement	True or False?	Proof, Counterexample or Explanation
$x^2 + y^2 = (x + y)^2$	F	Suppose that $x=1$ and $y=1$. Then L.H.S. = $1^2 + 1^2 = 2$ and R.H.S. = $(1+1)^2 = 2^2 = 4$ Since L.H.S. \neq R.H.S., the statement must be false. ✓
The equation $\frac{x^2}{16} + \frac{y^2}{9} = 0$ describes an ellipse.	F	Since $\frac{x^2}{16} \geq 0$ and $\frac{y^2}{9} \geq 0$ then the only possible solution to the given equation is $x=0$ and $y=0$. Therefore, this equation describes the single point $(0,0)$, <u>NOT</u> an ellipse. ✓
$\cos a + \cos b = \cos(a + b)$	F	Suppose that $a = \frac{\pi}{4}$ and $b = \frac{\pi}{4}$. Then, L.H.S. = $\cos \frac{\pi}{4} + \cos \frac{\pi}{4} = \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}} = \frac{2}{\sqrt{2}} = \sqrt{2}$. However, R.H.S. = $\cos(\frac{\pi}{4} + \frac{\pi}{4}) = \cos \frac{\pi}{2} = 0$. Since L.H.S. \neq R.H.S., the statement must be false. ✓
The distance between the points $P(x_1, y_1)$ and $Q(x_2, y_2)$ is given by the expression $\sqrt{\left(\frac{x_1 + x_2}{2}\right)^2 - \left(\frac{y_1 + y_2}{2}\right)^2}$	F	Consider the points $P(0,0)$ and $Q(1,0)$. Clearly, the length of PQ is <u>1 unit</u> . However, the given expression yields $\sqrt{\left(\frac{0+1}{2}\right)^2 - \left(\frac{0+0}{2}\right)^2} = \sqrt{\frac{1}{4}} = \frac{1}{2}$. Since $\frac{1}{2} \neq 1$, the statement must be false. ✓
The following transformations are applied to the function f : Vertical: Stretch by a factor of 3, then shift down 4 units. Horizontal: Compress by a factor of 2, then shift right 6 units. Then, the equation $y = 3f(2x - 6) - 4$ describes the transformed function.	F	The given equation correctly defines the vertical transformations. However, since the horizontal transformations require the "reversing" or "undoing" of the order of operations, the given equation actually defines a shift right by 6 units followed by a horizontal compression by a factor of 2. The correct equation is $y = 3f(2(x-6)) - 4$. ✓

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2. Complete the following statements by filling in the blanks with logical answers that relate to what we have learned in the review unit of this course. (7 KU)

Many students find mathematics difficult because they see it as set of symbols and rules that are applied without

justification and without understanding. To resolve these problems, Mr. Nolfi explained that math is like a dating service

because it's all about relationships. In addition, the terminology and notation of mathematics

can be seen as an extension of natural languages whose purpose is to

investigate and describe physical and conceptual relationships.

Mr. Nolfi also explained that it is a very bad idea to memorize formulas blindly (i.e. without understanding). Instead, he

suggested that we remember and understand some basic concepts that will help us to derive relationships quickly and

easily whenever necessary. For example,

for equations of lines, we only need to remember slope = slope,

for the midpoint of a line segment, we only need to remember how to find the average of two numbers

and for the length of a line segment, we only need to remember the Pythagorean theorem.

Finally, it is often helpful to understand physical interpretations of mathematical concepts. For instance, the concept of

slope is easier to grasp when we remember that it can be interpreted physically as steepness/velocity/rate of change

3. Simplify $3(2k-1)(-4k+7l-1) - (2k+l-3)^2$ (4 APP, 2 COM)

$$= (6k-3)(-4k+7l-1) - (2k+l-3)(2k+l-3)$$

$$= -24k^2 + 42kl - 6k + 12k - 21l + 3 - (4k^2 + 2kl - 6k + 2kl + l^2 - 3l - 6k - 3l + 9)$$

$$= -24k^2 + 42kl + 6k - 21l + 3 - 4k^2 - 4kl + 12k + 6l - l^2 - 9$$

$$= -28k^2 - l^2 + 38kl + 18k - 15l - 6$$

4. Fully factor $a^2(3a-2) - 5a(3a-2) + 9a-6$ (2 APP, 2 COM)

$$= a^2(3a-2) - 5a(3a-2) + 3(3a-2)$$

$$= (3a-2)(a^2 - 5a + 3)$$

This is a common factor

Note! This question should have been

$$\text{Factor } a^2(3a-2) - 4a(3a-2) + 9a-6$$

$$= a^2(3a-2) - 4a(3a-2) + 3(3a-2)$$

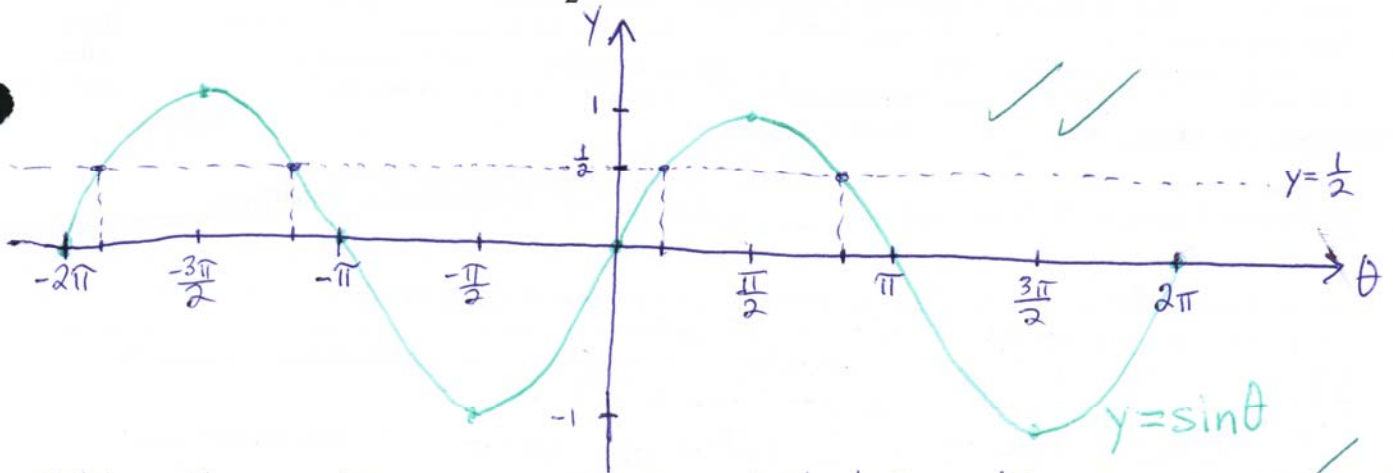
$$= (3a-2)(a^2 - 4a + 3)$$

$$= (3a-2)(a-1)(a-3)$$

Marks reduced due to an error in the question

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5. Use a graph to explain why the equation $\sin \theta = \frac{1}{2}$ must have exactly four solutions if $-2\pi \leq \theta \leq 2\pi$. (4 TIPS, 2 COM)



Solving the equation $\sin \theta = \frac{1}{2}$ is equivalent to asking the question, "For what values of θ is the y-value of $y = \sin \theta$ equal to $\frac{1}{2}$?" It is clear from the graph that $y = \sin \theta$ takes on the value $\frac{1}{2}$ at exactly four points within the range $-2\pi \leq \theta \leq 2\pi$. (This can also be interpreted as "Where does the graph of $y = \sin \theta$ intersect the graph of $y = \frac{1}{2}$?")

6. Solve $12\sin^2 x - 11\sin x + 2 = 0$ if $0 \leq x \leq 2\pi$. Express your answer(s) in radians. (5 APP, 3 COM)

$$12\sin^2 x - 11\sin x + 2 = 0$$

$$\therefore (4\sin x - 1)(3\sin x - 2) = 0$$

$$\therefore 4\sin x - 1 = 0 \text{ or } 3\sin x - 2 = 0$$

$$\therefore \sin x = \frac{1}{4} \text{ or } \sin x = \frac{2}{3}$$

$$\therefore x \doteq 0.253 \text{ rad}$$

$$\text{or } x \doteq 2.889 \text{ rad}$$

$$\therefore x \doteq 0.730 \text{ rad}$$

$$\text{or } x \doteq 2.412 \text{ rad}$$

Rough work!

Let $y = \sin x$. Then, the equation becomes

$$12y^2 - 11y + 2 = 0$$

$$\therefore 12y^2 - 8y - 3y + 2 = 0$$

$$\therefore 4y(3y - 2) - 1(3y - 2) = 0$$

$$\therefore (4y - 1)(3y - 2) = 0$$

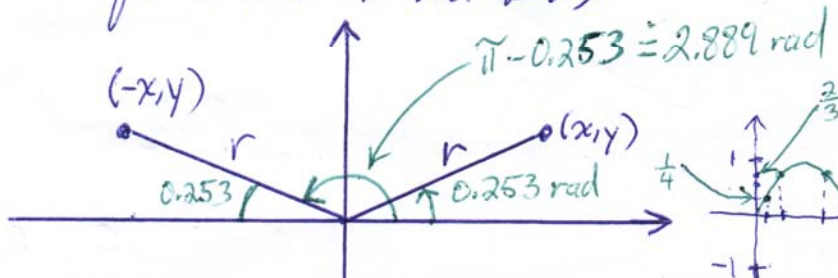
$$\therefore y = \frac{1}{4} \text{ or } y = \frac{2}{3}$$

Note: Here is an example of how to find the second answer once you obtain the first answer with your calculator. (Remember that your calculator will only give answers that lie in quadrants I and II.)

$$\sin(0.253) = \frac{y}{r}$$

$$\sin(2.889) = \frac{y}{r}$$

$$\therefore \sin(0.253) = \sin(2.889)$$



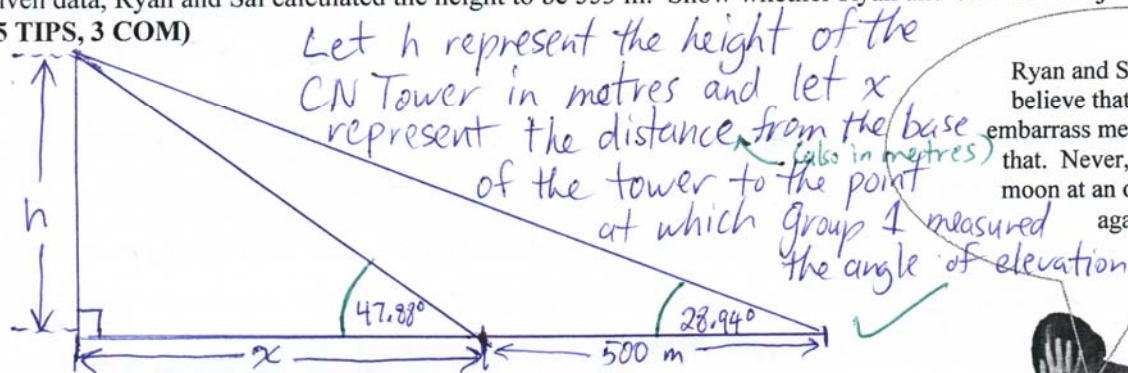
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7. Using her well honed interpersonal and managerial skills, Christine assembled two groups of MGA4U0 volunteer researchers to obtain an accurate measurement of the height of the CN Tower. The first group, consisting of Bhagat, Palwinder, Karan, A. J., Amandeep, Sehajpreet, Robbie, Stephanie and Andy, was told to measure the angle of elevation from the ground to the top of the CN tower. The second group, consisting of Manish, Snehjot, Philip, Gurdeep, Alex, Tapiwa, Harpreet, Antarpreet and Andrew were told to do the same from a point 500 m *more distant* from the tower. The results of the measurements are given in the following table.

Group 1: Bhagat, Palwinder, Karan, A. J., Amandeep, Sehajpreet, Robbie, Stephanie and Andy	Group 2: Manish, Snehjot, Philip, Gurdeep, Alex, Tapiwa, Harpreet, Antarpreet and Andrew
Angle of elevation: 47.88°	Angle of elevation: 28.94°

Fearing that Ryan and Sai might behave in an embarrassing manner in public, Christine decided to force them to stay at school. Once the two groups returned to school, they provided Ryan and Sai with the data that were collected. Using the given data, Ryan and Sai calculated the height to be 553 m. Show whether Ryan and Sai did their jobs correctly.

(5 TIPS, 3 COM)



Clearly,

$$\frac{h}{x} = \tan 47.88^\circ \quad \text{①} \quad \checkmark$$

$$\frac{h}{x+500} = \tan 28.94^\circ \quad \text{②} \quad \checkmark$$

From equation ① we have $x = \frac{h}{\tan 47.88^\circ}$. Substituting into equation ②, we obtain

$$h = \tan 28.94^\circ (x+500) = \tan 28.94^\circ \left(\frac{h}{\tan 47.88^\circ} + 500 \right)$$

$$\therefore h - \frac{h \tan 28.94^\circ}{\tan 47.88^\circ} = 500 \tan 28.94^\circ$$

$$\therefore h \left(1 - \frac{\tan 28.94^\circ}{\tan 47.88^\circ} \right) = 500 \tan 28.94^\circ$$

$$\therefore h = \frac{500 \tan 28.94^\circ}{1 - \frac{\tan 28.94^\circ}{\tan 47.88^\circ}}$$

$$\therefore h = 552.9$$

As difficult as it may be to believe, Ryan and Sai were correct!!

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