**MGA 4U0** **Semester 2, 2006 - 2007**

## Grade 12 Geometry and Discrete Mathematics
### Major Test – Unit 1

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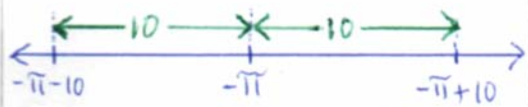
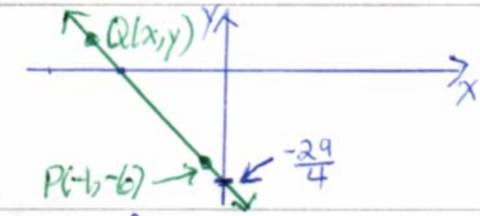
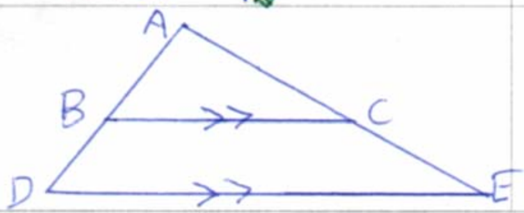
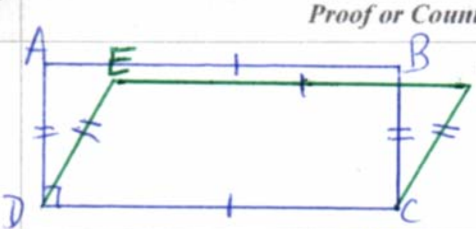
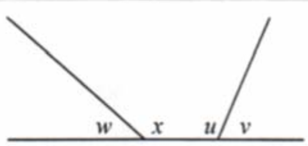
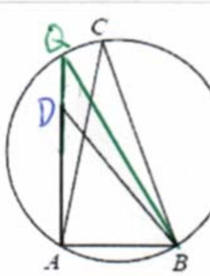
**Victim:** Mr. Solutions    Mr. S., you are amazing!!

| KU | APP | TIPS | COM |
|---|---|---|---|
| /22 | /16 | /15 | /19 |

1. Complete the following table. **(6 KU, 3 COM)**

| *Expression, Equation or Inequation* | *Diagram* | *Conclusion, Interpretation or Explanation* |
|---|---|---|
| $\lvert x+\pi\rvert=10$ <br> $\lvert x-(-\pi)\rvert=10$ |   $x=-\pi-10$ or $x=-\pi+10$ | The distance from $x$ to $-\pi$ is 10 units. |
| $\frac{y+6}{x+1}=-\frac{5}{4}$ <br> $y=-\frac{5}{4}x-\frac{29}{4}$ |   | The given equation describes a line with a slope of $-\frac{5}{4}$ and passing through $(-1,-6)$ |
| $BC \parallel DE$ if and only if $\frac{AB}{BD}=\frac{AC}{CE}$ |   | A line in a triangle is parallel to a side of the triangle *if and only if* it divides the other sides in the same proportion. |

2. State whether each of the following is true or false. To receive full credit, you must prove the statements that are true and provide a counterexample for the statements that are false. **(6 TIPS, 3 COM)**

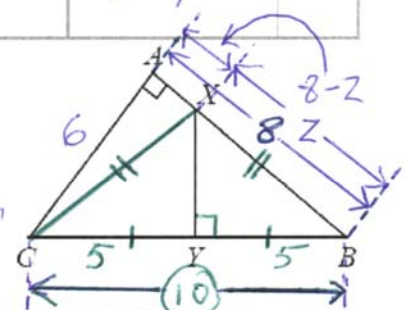
| *Statement* | *True or False?* | *Proof or Counterexample* |
|---|---|---|
| In quadrilaterals *ABCD* and *EFGH*, *AB*=*EF*, *BC*=*FG*, *CD*=*GH* and *DA*=*HE*. Therefore, $ABCD \cong EFGH$. | F |   Corresponding sides of the two quadrilaterals are equal but $ABCD \ncong EFCD$ |
|   $w+x+u+v=180°$ | F | $x+w=180°$ (SAT) <br> $u+v=180°$ (SAT) <br> $\therefore w+x+u+v=360°$ |
|   $\angle ACB > \angle ADB$ | F | Extend AD to Q in such a way that Q lies on the circle. <br> Then, $\angle AQB=\angle ACB$ (EASP) <br> Also, $\angle DAB+\angle DBA+\angle ADB=\angle QAB+\angle QBA+\angle AQB$ (ASTT) <br> Clearly, $\angle DAB=\angle QAB$. <br> Therefore, $\cancel{\angle QAB}+\angle DBA+\angle ADB=\cancel{\angle QAB}+\angle QBA+\angle ACB$ <br> $\therefore \angle DBA+\angle ADB=\angle QBA+\angle ACB$ |

Clearly, $\angle DBA < \angle QBA$
$\therefore \angle ADB > \angle ACB$ //

3. Complete the following table. In each column with the "T/F?" heading, indicate whether the corresponding statement is true or false. **(8 KU)**

| *Statement* ($P \rightarrow Q$) | *Statement in "If…then" form* | *T/F?* | *Converse* ($Q \rightarrow P$) | *T/F?* | *Inverse* ($\sim P \rightarrow \sim Q$) | *T/F?* | *Contrapositive* ($\sim Q \rightarrow \sim P$) | *T/F?* |
|---|---|---|---|---|---|---|---|---|
| $P$ implies $Q$ | If $P$ is true then $Q$ is true. | ? | If $Q$ is true then $P$ is true. | ? | If $P$ is not true then $Q$ is not true. | ? | If $Q$ is not true then $P$ is not true | ? |
| Ryan's cross-dressing alter-ego is Ryanna. | If the person is Ryan, then his cross-dressing alter-ego is Ryanna | T | If the cross-dressing alter-ego is Ryanna, the person must be Ryan. | F | If the person is not Ryan, then his/her cross-dressing alter-ego is not Ryanna | F | If the cross-dressing alter-ego is not Ryanna, the person is not Ryan | T |
| Philip has the best penmanship at Central Peel! | If the person is Philip, he has the best penmanship at CPSS. | F | If the person has the best penmanship at CPSS, the person must be Philip | F | If the person is not Philip, then he/she does not have the best penmanship at CPSS. | F | If the person does not have the best penmanship at CPSS, then he/she is not Philip. | F |

4. In right triangle $ABC$, $AB$=8 cm and $AC$=6 cm. By folding the triangle along line segment $XY$, vertex $B$ coincides with vertex $C$. What is the length of $XY$? **(4 APP)**




Construct $XC$

$\triangle XCY \cong \triangle XBY$ (since $\triangle XBY$ coincides with $\triangle XCY$ when folded along $XY$)

$\therefore XC = XB$

and $YC = YB = 5$

and $\angle CYX = \angle BYX = 90°$ (SAT and congruence)

$\therefore z^2 = 5^2 + XY^2$ (PT using $\triangle XBY$) (*)

and $z^2 = (8-z)^2 + 6^2$ (PT using $\triangle XAC$)

$\therefore z^2 = 64 - 16z + z^2 + 36$

$\therefore 16z = 100$

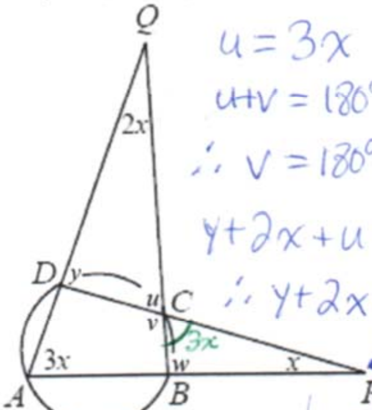
$\therefore z = \frac{100}{16} = \frac{25}{4}$

Substituting in equation (*) we obtain $XY = \frac{15}{4}$ cm. //

Blue/Black → given
Green → deduced
→ obtained using the Pythagorean Theorem

5. Determine the unknown values. (KU marks are given for stating the theorems used.)

(a) $x=\angle APD$, $y=\angle QDC$, $u=\angle QCD$, $v=\angle BCD$, $w=\angle CBP$ **(5 KU, 5 APP)**




$u = 3x$ (ACQP)

$u + v = 180°$ (SAT)

$\therefore v = 180° - u = 180° - 3x$

$y + 2x + u = 180°$ (ASTT)

$\therefore y + 2x + 3x = 180°$

$\therefore y = 180° - 5x$

$w = 2x + 3x$ (EAT)

$\therefore w = 5x$

Now $\angle CBP = 3x$ (ACQP)

In $\triangle CBP$

$w + 3x + x = 180°$ (ASTT)

$\therefore 5x + 3x + x = 180°$ (since $w = 5x$)

$\therefore 9x = 180°$

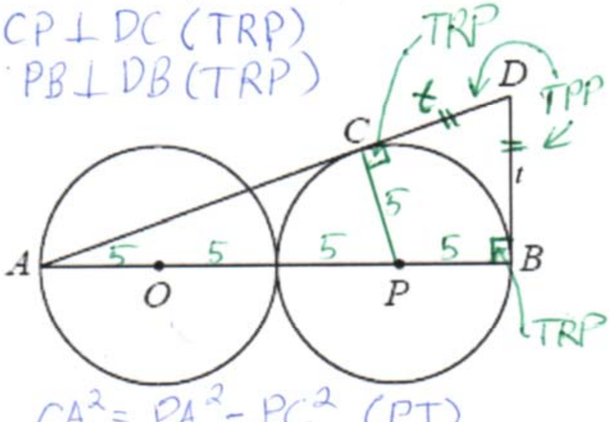
$\therefore x = 20°$

$\therefore u = 60°$, $v = 120°$, $y = 80°$ and $w = 100°$ //

(b) Given that the radius of each circle is 5 units, determine the length of $t=BD$. **(3 KU, 3 APP)**

$CP \perp DC$ (TRP)

$PB \perp DB$ (TRP)




$CA^2 = PA^2 - PC^2$ (PT)

$\therefore CA^2 = 15^2 - 5^2 = 200$

$\therefore CA = \sqrt{200} = 10\sqrt{2}$

$DC = DB = t$ (TPP)

Using right $\triangle DPA$

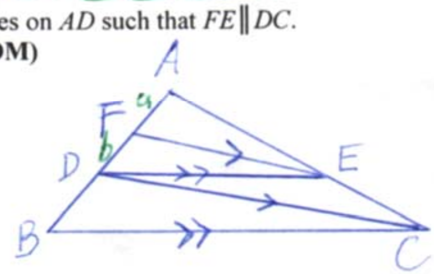
$t^2 + 20^2 = (t + 10\sqrt{2})^2$ (PT)

$\cancel{t^2} + 400 = \cancel{t^2} + 20\sqrt{2}t + 200$

$\therefore 20\sqrt{2}t = 200$

$t = \frac{200}{20\sqrt{2}} = \frac{10}{\sqrt{2}} = 5\sqrt{2}$ //

6. In $\Delta ABC$, $D$ and $E$ lie on $AB$ and $AC$ respectively such that $DE \parallel BC$. In addition, $F$ lies on $AD$ such that $FE \parallel DC$. Let $AF=a$ and $FD=b$. Determine the length of $BD$ in terms of $a$ and $b$. **(4 APP, 5 COM)**




In $\triangle ADC$,

$FE \parallel DC$

$\therefore \frac{AF}{FD} = \frac{a}{b} = \frac{AE}{EC}$ (TPPT)(*)

In $\triangle ABC$,

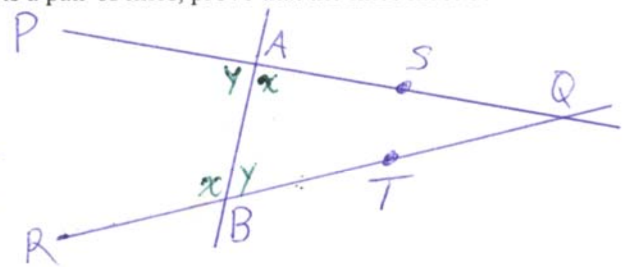
$DE \parallel BC$

$\therefore \frac{AD}{DB} = \frac{a+b}{DB} = \frac{AE}{EC}$ (TPPT)(**)

$\therefore \frac{a}{b} = \frac{a+b}{DB} = \frac{AE}{EC}$ (by (*) and (**))

$\therefore DB = \frac{b(a+b)}{a}$ //

7. In this question, you will use ***proof by contradiction*** (*reductio ad absurdum*) to prove one part of the "PLT Z" theorem. If alternate angles are equal when a transversal intersects a pair of lines, prove that the lines must be parallel. **(4 TIPS, 4 COM)**

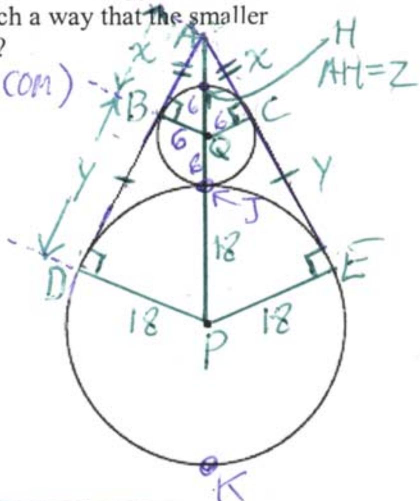



Suppose that
$\angle SAB = \angle RBA = x$ and
$\angle PAB = \angle TBA = y$ but $PS \not\parallel RT$.
Therefore, PS and PT must intersect at some point Q

Now $x + y = 180°$ (SAT) and
$x + y + \angle AQB = 180°$ (ASTT)

Therefore $\angle AQB = 0°$, which is impossible because the points A, Q and B form a triangle and each angle in a triangle must have a positive measure. Therefore, the original assumption that $PS \not\parallel RT$ is false. This means that $PS \parallel RT$. //

8. In a certain machine, an electric motor is used to spin a metal wheel of radius 6 cm, which is tangent to a larger metal wheel of radius 18 cm. A rubber belt is wrapped tightly around the two wheels in such a way that the smaller spinning wheel causes the larger wheel to spin. What is the length of the rubber belt? (5 TIPS, 4 COM)




Extend DB and EC to A. Then,
$AB = AC = x$ (TPP) and
$AD = AE$ (TPP)
$\therefore AD - AB = AE - AC$
$\therefore BD = CE = y$

Also,
$PD \perp AD, PE \perp AE, QB \perp AD, QC \perp AE$ (TRP)
$PD = PE = PJ = 18$ (radii of larger circle)
$QJ = QB = QC = QH = 6$ (radii of smaller circle)
$\triangle ABQ \sim \triangle ADP$ (AA)
$\therefore \frac{x}{x+y} = \frac{6}{18} = \frac{1}{3}$
$\therefore y = 2x$

continued

Also,
$\frac{AQ}{AP} = \frac{BQ}{DP} = \frac{1}{3}$
$\therefore \frac{z+6}{z+30} = \frac{1}{3}$
$\therefore z = 6$

In $\triangle ABQ$ (right triangle)
$AB^2 = AQ^2 - BQ^2$ (PT)
$\therefore x^2 = 12^2 - 6^2$
$\therefore x^2 = 108$
$\therefore x = \sqrt{108} = 6\sqrt{3}$

continued

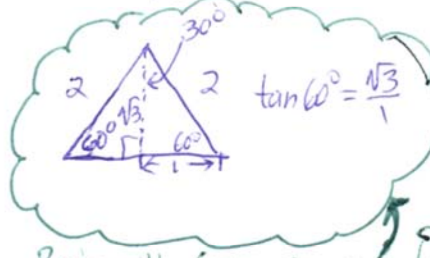
$\therefore y = 2x = 2(6\sqrt{3}) = 12\sqrt{3}$

$\tan(\angle APD) = \frac{AD}{PD} = \frac{6\sqrt{3} + 12\sqrt{3}}{18} = \frac{18\sqrt{3}}{18} = \sqrt{3}$

$\therefore \angle APD = 60^\circ$

$\therefore \angle DPE = 60^\circ + 60^\circ = 120^\circ$

$\therefore \angle BQC = 120^\circ$ (similar $\Delta$'s)




Begin with an equilateral $\Delta$ and then cut it in half.

(How to remember the $1:\sqrt{3}:2$ special triangle.)

$\therefore$ length of arc DKE

$= \frac{\cancel{240^\circ}^{2}}{\cancel{360^\circ}_{3}} \times \underbrace{2\pi(\cancel{18}_{6})}$ ← circumference of large circle

$= 24\pi$ units (cm)

and length of arc BHC

$= \frac{\cancel{120^\circ}^{1}}{\cancel{360^\circ}_{\cancel{3}\,1}} \times \underbrace{2\pi(\cancel{6}^{2})}$ ← circumference of smaller circle

$= 4\pi$ units (cm)

$\therefore$ length of rubber belt

$= BD + CE + \text{arc } BHC + \text{arc } DKE$

$= 2y + \text{arc } BHC + \text{arc } DKE$

$= 2(12\sqrt{3}) + 4\pi + 24\pi$

$= 24\sqrt{3} + 28\pi$ cm

$\doteq 129.5$ cm //