Ministry of Education
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Ontario

The Ontario Curriculum Grades 9 and 10

Mathematics

1999

Contents

Une publication équivalente est disponible en français sous le titre suivant : *Le curriculum de l'Ontario, 9e et 10e année – Mathématiques, 1999*.

This publication is available on the Ministry of Education and Training's World Wide Web site at http://www.edu.gov.on.ca.

Introduction

The Ontario Curriculum, Grades 9 and 10: Mathematics, 1999 will be implemented in Ontario secondary schools starting in September 1999 for students in Grade 9 and in September 2000 for students in Grade 10. This document replaces the sections in *The Common Curriculum: Policies and Outcomes, Grades 1–9, 1995* that relate to mathematics in Grade 9, and the parts of the following curriculum guidelines that relate to Grade 10:

– *Mathematics, Part One: Grades 9 and 10, Basic Level; Grades 11 and 12, Basic Level; Intermediate and Senior Divisions, 1985*

– *Mathematics, Part Two: Grades 7 and 8; Grades 9 and 10, General Level; Grades 11 and 12, General Level; Intermediate and Senior Divisions, 1985*

– *Mathematics, Part Three: Grades 7 and 8; Grades 9 and 10, Advanced Level; Grades 11 and 12, Advanced Level; Ontario Academic Courses; Intermediate and Senior Divisions, 1985*

This document is designed for use in conjunction with its companion piece, *The Ontario Curriculum, Grades 9 and 10: Program Planning and Assessment, 1999*, which contains information relevant to all disciplines represented in the curriculum. The planning and assessment document is available both in print and on the ministry's website, at http://www.edu.gov.on.ca.

The Place of Mathematics in the Curriculum

Unprecedented changes that are taking place in today's world will profoundly affect the futures of today's students. To meet the demands of the world in which they will live, students will need to adapt to changing conditions and to learn independently. They will require the ability to use technology effectively and the skills for processing large amounts of quantitative information. Today's mathematics curriculum must prepare students for their tomorrows. It must equip them with essential mathematical knowledge and skills; with skills of reasoning, problem solving, and communication; and, most importantly, with the ability and the incentive to continue learning on their own. This curriculum provides a framework for accomplishing these goals.

The choice of specific concepts and skills to be taught must take into consideration new applications and new ways of doing mathematics. The development of sophisticated yet easily used calculators and computers is changing the role of procedure and technique in mathematics. Operations that have been an essential part of a procedures-focused curriculum for decades can now be accomplished quickly and effectively using technology, so that students can now solve problems that were previously too time consuming to attempt, and can focus on underlying concepts. This curriculum integrates appropriate technologies into the learning and doing of mathematics, while recognizing the continuing importance of students' mastering essential arithmetic and algebraic skills.

Mathematical knowledge becomes meaningful and powerful in application. This curriculum embeds the learning of mathematics in the solving of problems based on real-life situations. Other disciplines are a ready source of effective contexts for the study of mathematics. Rich problem-solving situations can be drawn from closely related disciplines, such as computer science, physics, or technology, as well as from subjects historically thought of as distant from mathematics, such as geography or art. It is important that these links between disciplines be carefully explored, analysed, and discussed to emphasize for students the pervasiveness of mathematical knowledge and mathematical thinking in all subject areas.

Subject matter from any course in mathematics can be combined with subject matter from one or more courses in other disciplines to create an interdisciplinary course. The policies and procedures regarding the development of interdisciplinary courses are outlined in the interdisciplinary studies curriculum policy document.

The development of mathematical knowledge is a gradual process. A coherent and continuous program is necessary to help students see the "big pictures" or underlying principles of mathematics. The fundamentals of important skills, concepts, processes, and attitudes are initiated in the primary grades and fostered through elementary school. The links between Grade 8 and Grade 9 and the transition from elementary school mathematics to secondary school mathematics are very important in the student's development of confidence and competence.

The Grade 9 courses in this curriculum build on the concepts and skills expected of students by the end of Grade 8. The strands used are similar to those of the elementary program, with adjustments made to reflect the new directions that mathematics takes in secondary school. The philosophy of the Grade 9 courses is consistent with that of the elementary program and facilitates a seamless transition from elementary school, because it reflects the belief that students learn mathematics effectively when they have initial opportunities to explore through hands-on experiences, followed by careful guidance into an understanding of the abstract mathematics involved. Skill acquisition is an important part of the program; skills are embedded in the contexts offered by various topics in the mathematics program and should be introduced as they are needed.

The importance of communication in mathematics is a highlight of the elementary school curriculum and continues to be a highlight in secondary school. In all strands and all courses, expectations are included that require of students the clear and concise communication of reasoning or of findings. It is the expectation that students, over the course of their high school experience, will learn to write about their use of mathematics, effectively incorporating mathematical forms such as calculations, equations, graphs, or tables. This curriculum assumes a classroom environment in which students are called upon to explain their reasoning in writing, or orally to the teacher, to the class, or to other students in a group.

The Program in Mathematics

Overview

The material in each course of this curriculum is arranged by strands, which are major content organizers. It is expected that in developing detailed courses of study from this document, teachers will weave together related expectations from different strands, in order to create an overall program that integrates and balances concept development, skill acquisition, and applications.

An important part of every course in the mathematics program is the process of inquiry, in which students develop a systematic method for exploring new problems or unfamiliar situations. Described in specific expectations in both of the Grade 9 courses, the inquiry process is assumed as a major strategy that underlies teaching and learning at all levels and all grades. Knowing how to learn mathematics is the underlying expectation that every student in every course needs to achieve.

An important stage in the inquiry process is that of mathematical modelling, or taking the conditions of a real situation and describing them in mathematical form. A mathematical model can appear in many different ways – as an actual physical model, or as a diagram, a graph, a table of values, an equation, or a computer model. It is important that students understand the relationships among the various mathematical models of a given situation. To do so requires that their experiences in creating mathematical models increase in sophistication as they progress through high school. This curriculum is designed to encourage that growth. The process of inquiry is highlighted throughout the grades, but the sophistication and complexity of the problems, and the models that represent them, grow with the students.

The development of algebraic skills is a necessary feature of a balanced mathematics program. This curriculum has been designed to equip students with the algebraic skills necessary to understand other mathematics that they are learning, to solve meaningful problems, and to continue to learn mathematics with success in the future. The algebraic skills included in each course have been carefully chosen to support the other topics of that course. Calculators and other appropriate technology shall be used when the primary purpose of a given activity is the development of concepts or the solving of problems, or when a situation arises in which computation or symbolic manipulation is of secondary importance.

This curriculum is designed to make it possible for students to learn the mathematics they need for the destinations they seek, to foster the development of high levels of skill and knowledge.

In Grades 9 and 10, students may choose between two types of courses: *academic* and *applied*. (See *The Ontario Curriculum, Grades 9 and 10: Program Planning and Assessment, 1999* for a description of the different types of secondary school courses.) Courses offered in mathematics must be delivered as full courses, rather than as two half-credit courses.

Courses in Mathematics, Grades 9 and 10

Grade	Course Name	Course Type	Course Code	Credit Value	Prerequisites*
9	Principles of Mathematics	Academic	MPM1D	1	
9	Foundations of Mathematics	Applied	MFM1P	1	
10	Principles of Mathematics	Academic	MPM2D	1	Grade 9 Mathematics, Academic or Applied
10	Foundations of Mathematics	Applied	MFM2P	1	Grade 9 Mathematics, Academic or Applied

* Prerequisites apply only to Grade 10, 11, and 12 courses.

Teaching Approaches

Students in a mathematics class typically demonstrate diversity in the ways they best learn. It is important, therefore, that students have opportunities to learn in a variety of ways – individually, cooperatively, independently, with teacher direction, through hands-on experience, through examples followed by practice. The subject of mathematics varies in terms of the type of knowledge (concepts, skills, processes) that it contains, and competence in each type may be accomplished in different ways.

Because there is no single, correct way to teach or to learn mathematics, the nature of this curriculum demands that a variety of strategies be used in the classroom. It is assumed that the strategies used will vary according to the object of the learning and the needs of the students. Furthermore, content without process is ineffective, and process without content is meaningless. Rather, it is the integration of various aspects of mathematical knowledge that provides a powerful tool for reasoning and problem solving. This curriculum reflects a meaningful blend of both process and content.

All learning, especially new learning, should be embedded in a context. Well-chosen contexts for learning are those that are broad enough to allow students to explore and develop initial understandings, to identify and develop relevant supporting skills, and to gain experience with interesting applications of the new knowledge. Such rich environments open the door for students to see the "big ideas" of mathematics – the major underlying principles, such as pattern or relationship. This ability to understand key principles will encourage students to continue using mathematical reasoning throughout their lives.

Curriculum Expectations

The expectations identified for each course describe the knowledge and skills that students are expected to develop and demonstrate in their class work, on tests, and in various other activities on which their achievement is assessed and evaluated.

Two sets of expectations are listed for each strand, or broad curriculum area, of each course. The *overall expectations* describe in general terms the knowledge and skills that students are expected to demonstrate by the end of each course. The *specific expectations* describe the expected knowledge and skills in greater detail.

The specific expectations are organized under subheadings that correspond to the overall expectations. The specific expectations grouped under a subheading represent the detailed learning encompassed by the related overall expectation. This organization does not imply that the expectations in a particular group are to be achieved independently of those in any other group.

For some expectations, additional information is given in parentheses. Where the information begins with the abbreviation *i.e.*, it represents another way of expressing the expectation or provides a complete list of the details involved in the expectation. Where the information begins with the abbreviation *e.g.*, it comprises examples that are meant to illustrate the kind of skill, the specific area of learning, the depth of learning, and/or the level of complexity that the expectation entails. These examples are intended as a guide for teachers rather than as an exhaustive or mandatory list. Also provided in parentheses following some expectations are sample problems.

Strands

Grade 9

Course	Strands
Grade 9 Principles of Mathematics and Grade 9 Foundations of Mathematics	Number Sense and Algebra Relationships Analytic Geometry Measurement and Geometry

The strands of Grade 9 are designed to build upon those of Grade 8 while at the same time providing for growth in new directions in high school.

The strand Number Sense and Algebra builds on the Number Sense and Numeration strand of Grade 8. Throughout their experience in Grade 9 mathematics, students are expected to apply essential numeric skills in solving problems and in learning new material, thus taking opportunities to consolidate skills within a context, rather than practicing them in isolation. The strand also includes the algebraic skills necessary for the study and application of relations. These skills include the mastery of the basic exponent rules and of operations with first-degree polynomials, and the ability to solve first-degree equations.

Two strands – Relationships and Analytic Geometry – introduce the student to the geometry of the *xy*-plane. The focus of study in Grade 9 is the linear relation, with additional attention given to the study of non-linear relations. In the Relationships strand, students develop initial understandings of the properties of linear relations by collecting and interpreting data drawn from a variety of real-life situations and by creating algebraic models for the data. In the Analytic Geometry strand, the initial experiences of linear relations are then extended into the abstract realm of equations, formulas, and problems.

The strand Measurement and Geometry extends students' understandings from Grade 8 to include formulas for, and applications of, additional three-dimensional figures, but also involves students in explorations of relationships in measurement and in geometry. In measurement, this takes the form of investigations into the effect of varying one dimension, such as length, on a measure such as volume. Examination of the relationship between volume and surface area then leads to conclusions regarding the optimal size of three-dimensional objects, in applications such as packaging. In geometry, the students' Grade 8 knowledge of the properties of two-dimensional figures is extended through investigations that broaden their grasp and increase their intuitive understanding of the relationships among the properties. This understanding is an essential step in the development of strong spatial skills.

Grade 10

Course	Strands
Grade 10 Principles of Mathematics	Quadratic Functions Analytic Geometry Trigonometry
Grade 10 Foundations of Mathematics	Proportional Reasoning Linear Functions Quadratic Functions

The strands in the two Grade 10 courses have similarities, but there are significant differences between them in terms of level of abstraction and depth of complexity. Both courses contain a strand Quadratic Functions. The difference between the strands lies in the greater degree of algebraic treatment that occurs in the Principles course. Both strands involve concrete experiences upon which students build understandings of the abstract treatment of quadratic functions.

Both Grade 10 courses extend students' understandings of linear functions through applications, the Principles course doing so in the strand Analytic Geometry and the Foundations course, in the strand Linear Functions. While students in both courses study and apply linear systems, students in Grade 10 Foundations go on to examine applications of piecewise linear functions, and students in Grade 10 Principles solve multi-step problems involving the verification of properties of plane figures on the *xy*-plane. The topic of circles on the *xy*-plane is introduced in the Principles course as an application of the formula for the length of a line segment.

The third strand in the Principles course is called Trigonometry. In this strand, students apply trigonometry and the properties of similar triangles to solve problems in right and acute triangles. In the Foundations course, the strand that includes trigonometry is called Proportional Reasoning. Students deepen their understanding of proportional reasoning through a variety of topics involving, for example, similar triangles and trigonometry in right triangles.

Principles of Mathematics, Grade 9, Academic (MPM1D)

This course enables students to develop generalizations of mathematical ideas and methods through the exploration of applications, the effective use of technology, and abstract reasoning. Students will investigate relationships to develop equations of straight lines in analytic geometry, explore relationships between volume and surface area of objects in measurement, and apply extended algebraic skills in problem solving. Students will engage in abstract extensions of core learning that will deepen their mathematical knowledge and enrich their understanding.

Number Sense and Algebra

Overall Expectations

By the end of this course, students will:

- solve multi-step problems requiring numerical answers, using a variety of strategies and tools;
- demonstrate understanding of the three basic exponent rules and apply them to simplify expressions;
- manipulate first-degree polynomial expressions to solve first-degree equations;
- solve problems, using the strategy of algebraic modelling.

Specific Expectations

Solving Numerical Problems

By the end of this course, students will:

– demonstrate facility with critical numerical skills, including mental mathematics, estimation, operations with integers (as necessary for working with equations and analytic geometry), and operations with rational numbers (as necessary in analytic geometry, measurement, and equation solving);

– distinguish between exact and approximate representations of the same quantity and choose appropriately between them in given situations (e.g., use the symbol π instead of 3.14 in determining the effect on the volume of a sphere of doubling its diameter; determine the perimeter of a square having an area of 2);

– solve multi-step problems involving applications of percent, ratio, and rate as they arise throughout the course;

– use a scientific calculator effectively for applications that arise throughout the course;

– judge the reasonableness of answers to problems by considering likely results within the situation described in the problem;

– judge the reasonableness of answers produced by a calculator, a computer, or pencil and paper, using mental mathematics and estimation.

Operating with Exponents

By the end of this course, students will:

– evaluate numerical expressions involving natural-number exponents with rational-number bases;

– substitute into and evaluate algebraic expressions involving exponents, to support other topics of the course (e.g., measurement, analytic geometry);

– determine the meaning of negative exponents and of zero as an exponent from activities involving graphing, using technology, and from activities involving patterning;

– represent very large and very small numbers, using scientific notation;

– enter and interpret exponential notation on a scientific calculator, as necessary in calculations involving very large and very small numbers;

– determine, from the examination of patterns, the exponent rules for multiplying and dividing monomials and the exponent rule for the power of a power, and apply these rules in expressions involving one and two variables.

Manipulating Polynomial Expressions and Solving Equations

By the end of this course, students will:

– add and subtract polynomials;

– multiply a polynomial by a monomial, and factor a polynomial by removing a common factor;

– expand and simplify polynomial expressions involving one variable;

– solve first-degree equations, including equations with fractional coefficients, using an algebraic method;

– calculate sides in right triangles, using the Pythagorean theorem, as required in topics throughout the course (e.g., measurement);

– rearrange formulas involving variables in the first degree, with and without substitution, as they arise in topics throughout the course (e.g., analytic geometry, measurement).

Using Algebraic Modelling to Solve Problems

By the end of this course, students will:

– use algebraic modelling as one of several problem-solving strategies in various topics of the course (e.g., relations, measurement, direct and partial variation, the Pythagorean theorem, percent);

– compare algebraic modelling with other strategies used for solving the same problem;

– communicate solutions to problems in appropriate mathematical forms (e.g., written explanations, formulas, charts, tables, graphs) and justify the reasoning used in solving the problems.

Relationships

Overall Expectations

By the end of this course, students will:

- determine relationships between two variables by collecting and analysing data;
- compare the graphs and formulas of linear and non-linear relations;
- describe the connections between various representations of relations.

Specific Expectations

Determining Relationships

By the end of this course, students will:

– pose problems, identify variables, and formulate hypotheses associated with relationships (*Sample problem:* If you look through a paper tube at a wall, you can see a region of a certain height on the wall. If you move farther from the wall, the height of that region changes. What is the relationship between the height of the visible region and your distance from the wall? Describe the relationship that you think will occur);

– demonstrate an understanding of some principles of sampling and surveying (e.g., randomization, representivity, the use of multiple trials) and apply the principles in designing and carrying out experiments to investigate the relationships between variables (*Sample problem:* What factors might affect the outcome of this experiment? How could you design the experiment to account for them?);

– collect data, using appropriate equipment and/or technology (e.g., measuring tools, graphing calculators, scientific probes, the Internet) (*Sample problem:* Acquire or construct a paper tube and work with a partner to measure the heights of visible regions at various distances from a wall);

– organize and analyse data, using appropriate techniques (e.g., making tables and graphs, calculating measures of central tendency) and technology (e.g., graphing calculators, statistical software, spreadsheets) (*Sample problem:* Enter the data into a spreadsheet. Decide what analysis would be appropriate to examine the relationship between the variables – a graph, measures of central tendency, ratios);

– describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain the differences between the inferences and the hypotheses (*Sample problem:* Describe any trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your original hypothesis? Discuss any outlying pieces of data and provide explanations for them. Suggest a formula relating the height of the visible region to the distance from the wall. How might you vary this experiment to examine other relationships?);

– communicate the findings of an experiment clearly and concisely, using appropriate mathematical forms (e.g., written explanations, formulas, charts, tables, graphs), and justify the conclusions reached;

– solve and/or pose problems related to an experiment, using the findings of the experiment.

Comparing Linear and Non-linear Relations

By the end of this course, students will:

– construct tables of values, graphs, and formulas to represent linear relations derived from descriptions of realistic situations (e.g., the cost of holding a banquet in a rented hall is $25 per person plus $975 for the hall);

– construct tables of values and scatter plots for linearly related data collected from experiments (e.g., the rebound height of a ball versus the height from which it was dropped) or from secondary sources (e.g., the number of calories in fast food versus the number of grams of fat);

– determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., a process of trial and error on a graphing calculator; calculation of the equation of the line joining two carefully chosen points on the scatter plot);

– construct tables of values and graphs to represent non-linear relations derived from descriptions of realistic situations (*Sample problem*: A triangular prism has a height of 20 cm and a square base. Represent the relationship between the volume of the prism and the side length of its base, as the side length varies);

– construct tables of values and scatter plots for non-linearly related data collected from experiments (e.g., the relationship between height and age) or from secondary sources (e.g., the population of Canada over time); sketch a curve of best fit;

– demonstrate an understanding that straight lines represent linear relations and curves represent non-linear relations.

Describing Connections Between Representations of Relations

By the end of this course, students will:

– determine values of a linear relation by using the formula of the relation and by interpolating or extrapolating from the graph of the relation (e.g., if a student earns $5/h caring for children, determine how long he or she must work to earn $143);

– describe, in written form, a situation that would explain the events illustrated by a given graph of a relationship between two variables (e.g., write a story that matches the events shown in the graph);

– identify, by calculating finite differences in its table of values, whether a relation is linear or non-linear;

– describe the effect on the graph and the formula of a relation of varying the conditions of a situation they represent (e.g., if a graph showing partial variation represents the cost of producing a yearbook, describe how the appearance of the graph changes if the cost per book is altered; describe how it changes if the fixed costs are altered).

Analytic Geometry

Overall Expectations

By the end of this course, students will:

- determine, through investigation, the relationships between the form of an equation and the shape of its graph with respect to linearity and non-linearity;
- determine, through investigation, the properties of the slope and y-intercept of a linear relation;
- solve problems, using the properties of linear relations.

Specific Expectations

Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph

By the end of this course, students will:

– determine, through investigations, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use graphing software to obtain the graphs of a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; focus on the characteristics of the equations of linear relations and how they differ from the characteristics of the equations of non-linear relations);

– select the equations of straight lines from a given set of equations of linear and non-linear relations;

– identify the equation of a line in any of the forms $y = mx + b$, $Ax + By + C = 0$, $x = a$, $y = b$;

– rearrange the equation of a line from the form $y = mx + b$ to the form $Ax + By + C = 0$, and vice versa.

Investigating the Properties of Slope

By the end of this course, students will:

– determine the slope of a line segment, using various formulas (e.g., $m = \frac{\text{rise}}{\text{run}}$, $m = \frac{\Delta y}{\Delta x}$, $m = \frac{y_2 - y_1}{x_2 - x_1}$, $m = -\frac{A}{B}$);

– identify the slope of a linear relation as representing a constant rate of change;

– calculate the finite differences in the table of values of a linear relation and relate the result to the slope of the relation;

– identify the geometric significance of m and b in the equation $y = mx + b$ through investigation;

– identify the properties of the slopes of line segments (e.g., direction, positive or negative rate of change, steepness, parallelism, perpendicularity) through investigations facilitated by graphing technology, where appropriate.

Using the Properties of Linear Relations to Solve Problems

By the end of this course, students will:

– plot points on the *xy*-plane and use the terminology and notation of the *xy*-plane correctly;

– graph lines by hand, using a variety of techniques (e.g., making a table of values, using intercepts, using the slope and *y*-intercept);

– graph lines, using graphing calculators or graphing software;

– determine the equation of a line, given information about the line (e.g., the slope and *y*-intercept, the slope and a point, two points, a line parallel to a given line and having the same *x*-intercept as another given line);

– communicate solutions to multi-step problems in established mathematical form, with clear reasons given for the steps taken;

– describe the meaning of the slope and *y*-intercept for a linear relation arising from a realistic situation, interpolate and extrapolate from the graph and the equation of the relation, and identify and explain any restrictions on the variables in the relation;

– describe a situation that would be modelled by a given linear equation;

– determine the point of intersection of two linear relations, by hand for simple examples, and using graphing calculators or graphing software for more complex examples; interpret the intersection point in the context of an application.

Measurement and Geometry

Overall Expectations

By the end of this course, students will:

- determine the optimal values of various measurements through investigations facilitated, where appropriate, by the use of concrete materials, diagrams, and calculators or computer software;
- solve problems involving the surface area and the volume of three-dimensional objects;
- formulate conjectures and generalizations about geometric relationships involving two-dimensional figures, through investigations facilitated by dynamic geometry software, where appropriate.

Specific Expectations

Investigating the Optimal Value of Measurements

By the end of this course, students will:

– identify, through investigation, the effect of varying the dimensions of a rectangular prism or cylinder on the volume or surface area of the object;

– identify, through investigation, the relationships between the volume and surface area of a given rectangular prism or cylinder;

– explain the significance of optimal surface area or volume in various applications (e.g., packaging; the relationship between surface area and heat loss);

– pose and solve a problem involving the relationship between the perimeter and the area of a figure when one of the measures is fixed.

Solving Problems Involving Surface Area and Volume

By the end of this course, students will:

– solve simple problems, using the formulas for the surface area and the volume of prisms, pyramids, cylinders, cones, and spheres;

– solve multi-step problems involving the volume and the surface area of prisms, cylinders, pyramids, cones, and spheres;

– judge the reasonableness of answers to measurement problems by considering likely results within the situation described in the problem;

– judge the reasonableness of answers produced by a calculator, a computer, or pencil and paper, using mental mathematics and estimation.

Investigating Geometric Relationships

By the end of this course, students will:

– illustrate and explain the properties of the interior and the exterior angles of triangles and quadrilaterals, and of angles related to parallel lines;

– determine the properties of angle bisectors, medians, and altitudes in various types of triangles through investigation;

– determine the properties of the sides and the diagonals of polygons (e.g., the diagonals in quadrilaterals, the diagonals of regular pentagons, the figure that results from joining the midpoints of sides of quadrilaterals) through investigation;

– pose questions about geometric relationships, test them, and communicate the findings, using appropriate language and mathematical forms (e.g., written explanations, diagrams, formulas, tables);

– confirm a statement about the relationships between geometric properties by illustrating the statement with examples, or deny the statement on the basis of a counter-example (e.g., confirm or deny the following statement: If a quadrilateral has perpendicular diagonals, then it is a square).

Foundations of Mathematics, Grade 9, Applied (MFM1P)

This course enables students to develop mathematical ideas and methods through the exploration of applications, the effective use of technology, and extended experiences with hands-on activities. Students will investigate relationships of straight lines in analytic geometry, solve problems involving the measurement of 3-dimensional objects and 2-dimensional figures, and apply key numeric and algebraic skills in problem solving. Students will also have opportunities to consolidate core skills and deepen their understanding of key mathematical concepts.

Number Sense and Algebra

Overall Expectations

By the end of this course, students will:

- consolidate numerical skills by using them in a variety of contexts throughout the course;
- demonstrate understanding of the three basic exponent rules and apply them to simplify expressions;
- manipulate first-degree polynomial expressions to solve first-degree equations;
- solve problems, using the strategy of algebraic modelling.

Specific Expectations

Consolidating Numerical Skills

By the end of this course, students will:

– determine strategies for mental mathematics and estimation, and apply these strategies throughout the course;

– demonstrate facility in operations with integers, as necessary to support other topics of the course (e.g., polynomials, equations, analytic geometry);

– demonstrate facility in operations with percent, ratio and rate, and rational numbers, as necessary to support other topics of the course (e.g., analytic geometry, measurement);

– use a scientific calculator effectively for applications that arise throughout the course;

– judge the reasonableness of answers to problems by considering likely results within the situation described in the problem;

– judge the reasonableness of answers produced by a calculator, a computer, or pencil and paper, using mental mathematics and estimation.

Operating with Exponents

By the end of this course, students will:

– evaluate numerical expressions involving natural-number exponents with rational-number bases;

– substitute into and evaluate algebraic expressions involving exponents, to support other topics of the course (e.g., measurement, analytic geometry);

– determine the meaning of negative exponents and of zero as an exponent from activities involving graphing, using technology, and from activities involving patterning;

– represent very large and very small numbers, using scientific notation;

– enter and interpret exponential notation on a scientific calculator, as necessary in calculations involving very large and very small numbers;

– determine, from the examination of patterns, the exponent rules for multiplying and dividing monomials and the exponent rule for the power of a power, and apply these rules in expressions involving one variable.

Manipulating Polynomial Expressions and Solving Equations

By the end of this course, students will:

– add and subtract polynomials, and multiply a polynomial by a monomial;

– expand and simplify polynomial expressions involving one variable;

– solve first-degree equations, excluding equations with fractional coefficients, using an algebraic method;

– calculate sides in right triangles, using the Pythagorean theorem, as required in topics throughout the course (e.g., measurement);

– substitute into measurement formulas and solve for one variable, with and without the help of technology.

Using Algebraic Modelling to Solve Problems

By the end of this course, students will:

– use algebraic modelling as one of several problem-solving strategies in various topics of the course (e.g., relations, measurement, direct and partial variation, the Pythagorean theorem, percent);

– compare algebraic modelling with other strategies used for solving the same problem;

– communicate solutions to problems in appropriate mathematical forms (e.g., written explanations, formulas, charts, tables, graphs) and justify the reasoning used in solving the problems.

Relationships

Overall Expectations

By the end of this course, students will:

- determine relationships between two variables by collecting and analysing data;
- compare the graphs of linear and non-linear relations;
- describe the connections between various representations of relations.

Specific Expectations

Determining Relationships

By the end of this course, students will:

– pose problems, identify variables, and formulate hypotheses associated with relationships (*Sample problem:* Does the rebound height of a ball depend on the height from which it was dropped? Make a hypothesis and then design an experiment to test it);

– demonstrate an understanding of some principles of sampling and surveying (e.g., randomization, representivity, the use of multiple trials) and apply the principles in designing and carrying out experiments to investigate the relationships between variables (*Sample problem:* What factors might affect the outcome of this experiment? How could you design the experiment to account for them?);

– collect data, using appropriate equipment and/or technology (e.g., measuring tools, graphing calculators, scientific probes, the Internet) (*Sample problem:* Drop a ball from varying heights, measuring the rebound height each time);

– organize and analyse data, using appropriate techniques (e.g., making tables and graphs, calculating measures of central tendency) and technology (e.g., graphing calculators, statistical software, spreadsheets) (*Sample problem:* Enter the data into a spreadsheet. Decide what analysis would be appropriate to examine the relationship between the variables – a graph, measures of central tendency, ratios);

– describe trends and relationships observed in data, make inferences from data, compare the inferences with hypotheses about the data, and explain the differences between the inferences and the hypotheses (*Sample problem:* Describe any trend observed in the data. Does a relationship seem to exist? Of what sort? Is the outcome consistent with your original hypothesis? Discuss any outlying pieces of data and provide explanations for them. Suggest a formula relating the rebound height to the height from which the ball was dropped. How might you vary this experiment to examine other relationships?);

– communicate the findings of an experiment clearly and concisely, using appropriate mathematical forms (e.g., written explanations, formulas, charts, tables, graphs), and justify the conclusions reached;

– solve and/or pose problems related to an experiment, using the findings of the experiment.

Comparing Linear and Non-linear Relations

By the end of this course, students will:

- construct tables of values, graphs, and formulas to represent linear relations derived from descriptions of realistic situations involving direct and partial variation (e.g., the cost of holding a banquet in a rented hall is $25 per person plus $975 for the hall);
- construct tables of values and scatter plots for linearly related data involving direct variation collected from experiments (e.g., the rebound height of a ball versus the height from which it was dropped);
- determine the equation of a line of best fit for a scatter plot, using an informal process (e.g., a process of trial and error on a graphing calculator; calculation of the equation of the line joining two carefully chosen points on the scatter plot);
- construct tables of values and graphs to represent non-linear relations derived from descriptions of realistic situations (e.g., represent the relationship between the volume of a cube and its side length, as the side length varies);
- demonstrate an understanding that straight lines represent linear relations and curves represent non-linear relations.

Describing Connections Between Representations of Relations

By the end of this course, students will:

- determine values of a linear relation by using the formula of the relation and by interpolating or extrapolating from the graph of the relation (e.g., if a student earns $5/h caring for children, determine how long he or she must work to earn $143);
- describe, in written form, a situation that would explain the events illustrated by a given graph of a relationship between two variables (e.g., write a story that matches the events shown in the graph);
- identify, by calculating finite differences in its table of values, whether a relation is linear or non-linear;
- describe the effect on the graph and the formula of a relation of varying the conditions of a situation they represent (e.g., if a graph showing partial variation represents the cost of producing a yearbook, describe how the appearance of the graph changes if the cost per book is altered; describe how it changes if the fixed costs are altered).

Analytic Geometry

Overall Expectations

By the end of this course, students will:

- determine, through investigation, the relationships between the form of an equation and the shape of its graph with respect to linearity and non-linearity;
- determine, through investigation, the properties of the slope and y-intercept of a linear relation;
- graph a line and write the equation of a line from given information.

Specific Expectations

Investigating the Relationship Between the Equation of a Relation and the Shape of Its Graph

By the end of this course, students will:

– determine, through investigations, the characteristics that distinguish the equation of a straight line from the equations of non-linear relations (e.g., use graphing software to obtain the graphs of a variety of linear and non-linear relations from their equations; classify the relations according to the shapes of their graphs; focus on the characteristics of the equations of linear relations and how they differ from the characteristics of the equations of non-linear relations);

– select the equations of straight lines from a given set of equations of linear and non-linear relations;

– identify $y = mx + b$ as a standard form for the equation of a straight line, including the special cases $x = a$, $y = b$.

Investigating the Properties of Slope

By the end of this course, students will:

– identify practical situations illustrating slope (e.g., ramps, slides, staircases) and calculate the slopes of the inclines;

– determine the slope of a line segment, using the formula $m = \frac{\text{rise}}{\text{run}}$;

– identify the geometric significance of m and b in the equation $y = mx + b$ through investigation;

– identify the properties of the slopes of line segments (i.e., direction, positive or negative rate of change, steepness, parallelism, perpendicularity) through investigations facilitated by graphing technology, where appropriate.

Graphing and Writing Equations of Lines

By the end of this course, students will:

– plot points on the xy-plane and use the terminology and notation of the xy-plane correctly;

– graph lines by hand, using a variety of techniques (e.g., making a table of values, using intercepts, using the slope and y-intercept);

– graph lines, using graphing calculators or graphing software;

– determine the equation of a line, given the slope and y-intercept, the slope and a point on the line, and two points on the line;

– communicate solutions in established mathematical form, with clear reasons given for the steps taken.

Measurement and Geometry

Overall Expectations

By the end of this course, students will:

- determine the optimal values of various measurements through investigations facilitated by the use of concrete materials, diagrams, and calculators or computer software;
- solve problems involving the measurement of two-dimensional figures and three-dimensional objects;
- formulate conjectures and generalizations about geometric relationships involving two-dimensional figures, through investigations facilitated by dynamic geometry software, where appropriate.

Specific Expectations

Investigating the Optimal Values of Measurements

By the end of this course, students will:

– construct a variety of rectangles for a given perimeter and determine the maximum area for a given perimeter;

– construct a variety of square-based prisms for a given volume and determine the minimum surface area for a square-based prism with a given volume;

– construct a variety of cylinders for a given volume and determine the minimum surface area for a cylinder with a given volume;

– describe applications in which it would be important to know the maximum area for a given perimeter or the minimum surface area for a given volume (e.g., building a fence, designing a container).

Solving Problems Involving Measurement

By the end of this course, students will:

– solve problems involving the area of composite plane figures (e.g., combinations of rectangles, triangles, parallelograms, trapezoids, and circles);

– solve simple problems, using the formulas for the surface area of prisms and cylinders and for the volume of prisms, cylinders, cones, and spheres;

– solve problems involving perimeter, area, surface area, volume, and capacity in applications;

– judge the reasonableness of answers to measurement problems by considering likely results within the situation described in the problem;

– judge the reasonableness of answers produced by a calculator, a computer, or pencil and paper, using mental mathematics and estimation.

Investigating Geometric Relationships

By the end of this course, students will:

– illustrate and explain the properties of the interior and exterior angles of triangles and quadrilaterals, and of angles related to parallel lines;

– determine the properties of angle bisectors, medians, and altitudes in various types of triangles through investigation;

– determine some properties of the sides and the diagonals of quadrilaterals (e.g., the diagonals of a rectangle bisect each other);

– communicate the findings of investigations, using appropriate language and mathematical forms (e.g., written explanations, diagrams, formulas, tables).

Principles of Mathematics, Grade 10, Academic (MPM2D)

This course enables students to broaden their understanding of relations, extend their skills in multi-step problem solving, and continue to develop their abilities in abstract reasoning. Students will pursue investigations of quadratic functions and their applications; solve and apply linear systems; solve multi-step problems in analytic geometry to verify properties of geometric figures; investigate the trigonometry of right and acute triangles; and develop supporting algebraic skills.

Quadratic Functions

Overall Expectations

By the end of this course, students will:

- solve quadratic equations;
- determine, through investigation, the relationships between the graphs and the equations of quadratic functions;
- determine, through investigation, the basic properties of quadratic functions;
- solve problems involving quadratic functions.

Specific Expectations

Solving Quadratic Equations

By the end of this course, students will:

– expand and simplify second-degree polynomial expressions;

– factor polynomial expressions involving common factors, differences of squares, and trinomials;

– solve quadratic equations by factoring and by using graphing calculators or graphing software;

– solve quadratic equations, using the quadratic formula;

– interpret real and non-real roots of quadratic equations geometrically as the x-intercepts of the graph of a quadratic function.

Investigating the Connection Between the Graphs and the Equations of Quadratic Functions

By the end of this course, students will:

– identify the effect of simple transformations (i.e., translations, reflections, vertical stretch factors) on the graph and the equation of $y = x^2$, using graphing calculators or graphing software;

– explain the role of a, h, and k in the graph of $y = a(x - h)^2 + k$;

– express the equation of a quadratic function in the form $y = a(x - h)^2 + k$, given it in the form $y = ax^2 + bx + c$, using the algebraic method of completing the square in situations involving no fractions;

– sketch, by hand, the graph of a quadratic function whose equation is given in the form $y = ax^2 + bx + c$, using a suitable method [e.g., complete the square; locate the x-intercepts if the equation is factorable; express in the form $y = ax(x - s) + t$ to locate two points and deduce the vertex].

Investigating the Basic Properties of Quadratic Functions

By the end of this course, students will:

– collect data that may be represented by quadratic functions, from secondary sources (e.g., the Internet, Statistics Canada), or from experiments, using appropriate equipment and technology (e.g., scientific probes, graphing calculators);

– fit the equation of a quadratic function to a scatter plot, using an informal process (e.g., a process of trial and error on a graphing calculator), and compare the results with the equation of a curve of best fit produced by using graphing calculators or graphing software;

– describe the nature of change in a quadratic function, using finite differences in tables of values, and compare the nature of change in a quadratic function with the nature of change in a linear function;
– report the findings of an experiment in a clear and concise manner, using appropriate mathematical forms (e.g., written explanations, tables, graphs, formulas, calculations), and justify the conclusions reached.

Solving Problems Involving Quadratic Functions

By the end of this course, students will:

– determine the zeros and the maximum or minimum value of a quadratic function, using algebraic techniques;
– determine the zeros and the maximum or minimum value of a quadratic function from its graph, using graphing calculators or graphing software;
– solve problems related to an application, given the graph or the formula of a quadratic function (e.g., given a quadratic function representing the height of a ball over elapsed time, answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball touch the ground? Over what interval is the height of the ball greater than 3 m?).

Analytic Geometry

Overall Expectations

By the end of this course, students will:

- model and solve problems involving the intersection of two straight lines;
- solve problems involving the analytic geometry concepts of line segments;
- verify geometric properties of triangles and quadrilaterals, using analytic geometry.

Specific Expectations

Using Linear Systems to Solve Problems

By the end of this course, students will:

– determine the point of intersection of two linear relations graphically, with and without the use of graphing calculators or graphing software, and interpret the intersection point in the context of a realistic situation;

– solve systems of two linear equations in two variables by the algebraic methods of substitution and elimination;

– solve problems represented by linear systems of two equations in two variables arising from realistic situations, by using an algebraic method and by interpreting graphs.

Solving Problems Involving the Properties of Line Segments

By the end of this course, students will:

– determine formulas for the midpoint and the length of a line segment and use these formulas to solve problems;

– determine the equation for a circle having centre (0, 0) and radius r, by applying the formula for the length of a line segment; identify the radius of a circle of centre (0,0), given its equation; and write the equation, given the radius;

– solve multi-step problems, using the concepts of the slope, the length, and the midpoint of line segments (e.g., determine the equation of the right bisector of a line segment, the coordinates of whose end points are given; determine the distance from a given point to a line whose equation is given; show that the centre of a given circle lies on the right bisector of a given chord);

– communicate the solutions to multi-step problems in good mathematical form, giving clear reasons for the steps taken to reach the solutions.

Using Analytic Geometry to Verify Geometric Properties

By the end of this course, students will:

– determine characteristics of a triangle whose vertex coordinates are given (e.g., the perimeter; the classification by side length; the equations of medians, altitudes, and right bisectors; the location of the circumcentre and the centroid);

– determine characteristics of a quadrilateral whose vertex coordinates are given (e.g., the perimeter; the classification by side length; the properties of the diagonals; the classification of a quadrilateral as a square, a rectangle, or a parallelogram);

– verify geometric properties of a triangle or quadrilateral whose vertex coordinates are given (e.g., the line joining the midpoints of two sides of a triangle is parallel to the third side; the diagonals of a rectangle bisect each other).

Trigonometry

Overall Expectations

By the end of this course, students will:

- develop the primary trigonometric ratios, using the properties of similar triangles;
- solve trigonometric problems involving right triangles;
- solve trigonometric problems involving acute triangles.

Specific Expectations

Developing the Primary Trigonometric Ratios

By the end of this course, students will:

– determine the properties of similar triangles (e.g., the correspondence and equality of angles, the ratio of corresponding sides, the ratio of areas) through investigation, using dynamic geometry software;

– describe and compare the concepts of similarity and congruence;

– solve problems involving similar triangles in realistic situations (e.g., problems involving shadows, reflections, surveying);

– define the formulas for the sine, the cosine, and the tangent of angles, using the ratios of sides in right triangles.

Solving Problems Involving the Trigonometry of Right Triangles

By the end of this course, students will:

– determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios;

– solve problems involving the measures of sides and angles in right triangles (e.g., in surveying, navigation);

– determine the height of an inaccessible object in the environment around the school, using the trigonometry of right triangles.

Solving Problems Involving the Trigonometry of Acute Triangles

By the end of this course, students will:

– determine, through investigation, the relationships between the angles and sides in acute triangles (e.g., the largest angle is opposite the longest side; the ratio of side lengths is equal to the ratio of the sines of the opposite angles), using dynamic geometry software;

– calculate the measures of sides and angles in acute triangles, using the sine law and cosine law;

– describe the conditions under which the sine law or the cosine law should be used in a problem;

– solve problems involving the measures of sides and angles in acute triangles;

– describe the application of trigonometry in science or industry.

Foundations of Mathematics, Grade 10, Applied (MFM2P)

This course enables students to consolidate their understanding of key mathematical concepts through hands-on activities and to extend their problem-solving experiences in a variety of applications. Students will solve problems involving proportional reasoning and the trigonometry of right triangles; investigate applications of piecewise linear functions; solve and apply systems of linear equations; and solve problems involving quadratic functions. The effective use of technology in learning and in solving problems will be a focus of the course.

Proportional Reasoning

Overall Expectations

By the end of this course, students will:

- solve problems derived from a variety of applications, using proportional reasoning;
- solve problems involving similar triangles;
- solve problems involving right triangles, using trigonometry.

Specific Expectations

Using Proportional Reasoning to Solve Problems from Applications

By the end of this course, students will:

- solve problems involving percent, ratio, rate, and proportion (e.g., in topics such as interest calculation, currency conversion, similar triangles, trigonometry, direct and partial variation related to linear functions) by a variety of methods and models (e.g., diagrams, concrete materials, fractions, tables, patterns, graphs, equations);
- draw and interpret scale diagrams related to applications (e.g., technical drawings);
- distinguish between consistent and inconsistent representations of proportionality in a variety of contexts (e.g., explain the distortion of figures resulting from irregular scales; identify misleading features in graphs; identify misleading conclusions based on invalid proportional reasoning).

Solving Problems Involving Similar Triangles

By the end of this course, students will:

- determine some properties of similar triangles (e.g., the correspondence and equality of angles, the ratio of corresponding sides) through investigation, using dynamic geometry software;
- solve problems involving similar triangles in realistic situations (e.g., problems involving shadows, reflections, surveying);
- define the formulas for the sine, the cosine, and the tangent of angles, using the ratios of sides in right triangles.

Solving Problems Involving the Trigonometry of Right Triangles

By the end of this course, students will:

- calculate the length of a side of a right triangle, using the Pythagorean theorem;
- determine the measures of the sides and angles in right triangles, using the primary trigonometric ratios;
- solve problems involving the measures of sides and angles in right triangles (e.g., in surveying, navigation);
- determine the height of an inaccessible object in the environment around the school, using the trigonometry of right triangles;
- describe applications of trigonometry in various occupations.

Linear Functions

Overall Expectations

By the end of this course, students will:

- apply the properties of piecewise linear functions as they occur in realistic situations;
- solve and interpret systems of two linear equations as they occur in applications;
- manipulate algebraic expressions as they relate to linear functions.

Specific Expectations

Applying Piecewise Linear Functions

By the end of this course, students will:

– explain the characteristics of situations involving piecewise linear functions (e.g., pay scale variations, gas consumption costs, water consumption costs, differentiated pricing, motion);

– construct tables of values and sketch graphs to represent given descriptions of realistic situations involving piecewise linear functions, with and without the use of graphing calculators or graphing software;

– answer questions about piecewise linear functions by interpolation and extrapolation, and by considering variations on given conditions.

Interpreting Systems of Linear Equations

By the end of this course, students will:

– determine the point of intersection of two linear relations arising from a realistic situation, using graphing calculators or graphing software;

– interpret the point of intersection of two linear relations within the context of a realistic situation;

– solve systems of two linear equations in two variables by the algebraic methods of substitution and elimination;

– solve problems represented by linear systems of two equations in two variables arising from realistic situations, by using an algebraic method and by interpreting graphs.

Manipulating Algebraic Expressions

By the end of this course, students will:

– write linear equations by generalizing from tables of values and by translating written descriptions;

– rearrange equations from the form $y = mx + b$ to the form $Ax + By + C = 0$, and vice versa;

– solve first-degree equations in one variable, including those with fractional coefficients, using an algebraic method;

– isolate a variable in formulas involving first-degree terms.

Quadratic Functions

Overall Expectations

By the end of this course, students will:

- manipulate algebraic expressions as they relate to quadratic functions;
- determine, through investigation, the relationships between the graphs and the equations of quadratic functions;
- solve problems by interpreting graphs of quadratic functions.

Specific Expectations

Manipulating Algebraic Expressions

By the end of this course, students will:

- multiply two binomials and square a binomial;
- expand and simplify polynomial expressions involving the multiplying and squaring of binomials;
- describe intervals on quadratic functions, using appropriate vocabulary (e.g., *greater than*, *less than*, *between*, *from ... to*, *less than 3 or greater than 7*);
- factor polynomials by determining a common factor;
- factor trinomials of the form $x^2 + bx + c$;
- factor the difference of squares;
- solve quadratic equations by factoring.

Investigating the Connection Between the Graphs and the Equations of Quadratic Functions

By the end of this course, students will:

- construct tables of values, sketch graphs, and write equations of the form $y = ax^2 + b$ to represent quadratic functions derived from descriptions of realistic situations (e.g., vary the side length of a cube and observe the effect on the surface area of the cube);
- identify the effect of simple transformations (i.e., translations, reflections, vertical stretch factors) on the graph and the equation of $y = x^2$, using graphing calculators or graphing software;
- explain the role of a, h, and k in the graph of $y = a(x - h)^2 + k$;
- expand and simplify an equation of the form $y = a(x - h)^2 + k$ to obtain the form $y = ax^2 + bx + c$.

Solving Problems Involving Quadratic Functions

By the end of this course, students will:

- obtain the graphs of quadratic functions whose equations are given in the form $y = a(x - h)^2 + k$ or the form $y = ax^2 + bx + c$, using graphing calculators or graphing software;
- determine the zeros and the maximum or minimum value of a quadratic function from its graph, using graphing calculators or graphing software;
- solve problems involving a given quadratic function by interpreting its graph (e.g., given a formula representing the height of a ball over elapsed time, graph the function, using a graphing calculator or graphing software, and answer questions such as the following: What is the maximum height of the ball? After what length of time will the ball touch the ground? Over what interval is the height of the ball greater than 3 m?).

Some Considerations for Program Planning in Mathematics

Teachers who are planning a program in mathematics must take into account considerations in a number of important areas. Essential information that pertains to all disciplines is provided in the companion piece to this document, *The Ontario Curriculum, Grades 9 and 10: Program Planning and Assessment, 1999*. The areas of concern to all teachers that are outlined there include the following:

- types of secondary school courses
- education for exceptional students
- the role of technology in the curriculum
- English as a second language (ESL) and English literacy development (ELD)
- career education
- cooperative education and other workplace experiences
- health and safety

Additional considerations that have particular relevance for program planning in mathematics are noted here.

Education for Exceptional Students. Exceptional students may require program modifications that specifically address their strengths and needs in learning mathematics. Some students will benefit from additional time to internalize important mathematical concepts. In an effort to help students understand concepts, teachers may include more concrete experiences, vary the pace of the learning, use additional examples, or insert cumulative reviews. The appropriate use of technologies (e.g., concrete materials, dynamic geometry software) will support the teacher in meeting the needs of exceptional students as set out in their Individual Education Plan.

The Role of Technology in the Curriculum. Technology helps to make students more powerful learners by giving them the means to explore mathematical concepts more easily and quickly. In the time gained by using technology, students can study fundamental ideas in greater depth, and can concentrate their effort in the areas of data collection, data analysis, simulations, and complex problem solving. Whereas student investigators once relied solely on their creativity and their sophistication in the use of largely paper-and-pencil methods to guide them in the solution of problems, they can now turn to technology, which provides capabilities that alter both the form and the means of solution.

The use of spreadsheets and symbolic manipulators permits students to explore without excess algebraic manipulation. Calculators save students time in performing complex arithmetic calculations. Graphing utilities enable students to visualize relationships and test hypotheses. Statistical packages and scientific probes allow students to collect and analyse real data from real experiments. The use of technology in learning and doing mathematics also gives students excellent opportunities to develop their abilities in algorithmic thinking, for example, by creating templates in spreadsheets or by writing sequences of instructions in application programs as part of a problem-solving process.

The presence of technology as part of learning mathematics makes many new things possible, but it also places an increasing importance on the ability of students to make mental judgements about expected results. For example, the student who uses a calculator to perform an arithmetic calculation should have the habit of using estimation to judge the reasonableness of the answer produced. Similarly, the student who produces a graph using technology should be capable of creating a mental approximation of the graph as a verification of the image on the screen.

English As a Second Language and English Literacy Development (ESL/ELD). There are several key considerations for teachers of mathematics in planning programs for ESL/ELD students. Teachers must recognize the mathematical skills and knowledge that all students bring to the classroom, and should value the students' knowledge and build on their strengths. Teachers should approach with sensitivity the increased emphasis on communication in mathematics, especially in cooperative learning settings, so that difficulties with language do not inhibit the participation of ESL/ELD students and hinder their success. Students should be encouraged to communicate their thoughts and understandings about mathematics in oral and written form, using the language of mathematics. ESL/ELD students should be encouraged to discuss problems, justify reasons for answers, and compare ideas and strategies within the classroom. Teachers must ensure that reading levels are appropriate to students' abilities and must strive for clarity in the use of mathematical terminology. Where possible, teachers should use visual and interactive methods, including technology, to facilitate the learning of mathematics, and should make appropriate accommodations and modifications for assessment.

Career Education. Teachers should promote students' understanding of the role of mathematics in daily life and its relation to career opportunities by exploring applications of concepts, providing opportunities for career-related project work, and promoting independent investigations. Such activities allow students the opportunity to investigate mathematics-related careers compatible with their interests, aspirations, and abilities.

Cooperative Education and Other Workplace Experiences. For students in mathematics, the hands-on experience of work and of cooperative education can reveal career options previously unknown to them and can provide valuable lessons on the application of mathematics in work and society. Cooperative placements in many occupations (e.g., engineering, accounting, banking, dentistry, the skilled trades, teaching, landscape design, computer programming, medicine, and fashion design) can give students greater insight into the value of learning mathematics.

Mathematics Anxiety. Mathematics anxiety is a state of mind relating to a student's perception of his or her ability to do mathematics. It is neither grade specific nor exclusively gender related. If left unchecked, it often leads to mathematics avoidance. To alleviate this anxiety in classrooms, teachers should:

– be accepting, patient, and understanding;
– defuse tense situations if they arise;
– make mathematics relevant by connecting the context with the student's life experience;
– provide many opportunities for students to be successful;
– set up programs for peer tutoring;
– use a variety of assessment techniques (journals, interviews, portfolios, projects);
– comment positively on material that is assessed;
– be aware of cultural biases.

Much mathematics anxiety can be avoided by using good teaching techniques. Being aware of situations that can cause tension and working with students in a caring, understanding, and encouraging way can do much to alleviate anxiety.

The Achievement Chart for Mathematics

The achievement chart that follows identifies four categories of knowledge and skills in mathematics – Knowledge/Understanding, Thinking/Inquiry/Problem Solving, Communication, and Application. These categories encompass all the curriculum expectations in courses in the discipline. For each of the category statements in the left-hand column, the levels of student achievement are described. (Detailed information on the achievement levels and on assessment, evaluation, and reporting policy is provided in *The Ontario Curriculum, Grades 9 and 10: Program Planning and Assessment, 1999*.)

The achievement chart is meant to guide teachers in:
- planning instruction and learning activities that will lead to the achievement of the curriculum expectations in a course;
- planning assessment strategies that will accurately assess students' achievement of the curriculum expectations;
- selecting samples of student work that provide evidence of achievement at particular levels;
- providing descriptive feedback to students on their current achievement and suggesting strategies for improvement;
- determining, towards the end of a course, the student's most consistent level of achievement of the curriculum expectations as reflected in his or her course work;
- devising a method of final evaluation;
- assigning a final grade.

The achievement chart can guide students in:
- assessing their own learning;
- planning strategies for improvement, with the help of their teachers.

The achievement chart provides a standard province-wide method for teachers to use in assessing and evaluating their students' achievement. Teachers will be provided with materials that will assist them in improving their assessment methods and strategies and, hence, their assessment of student achievement. These materials will contain samples of student work (exemplars) that illustrate achievement at each of the levels (represented by associated percentage grade ranges). Until these materials are provided, teachers may continue to follow their current assessment and evaluation practices.

To ensure consistency in assessment and reporting across the province, the ministry will provide samples of student work that reflect achievement based on the provincial standard, and other resources based on the achievement charts. As these resources become available, teachers will begin to use the achievement charts in their assessment and evaluation practices.

To support this process, the ministry will provide the following:
- a standard provincial report card, with an accompanying guide
- course profiles
- exemplars
- curriculum and assessment videos
- training materials
- an electronic curriculum planner

When planning courses and assessment, teachers should review the required curriculum expectations and link them to the categories to which they relate. They should ensure that all the expectations are accounted for in instruction, and that achievement of the expectations is assessed within the appropriate categories. The descriptions of the levels of achievement given in the chart should be used to identify the level at which the student has achieved the expectations. Students should be given numerous and varied opportunities to demonstrate their achievement of the expectations across the four categories. Teachers may find it useful to provide students with examples of work at the different levels of achievement.

The descriptions of achievement at level 3 reflect the provincial standard for student achievement. A complete picture of overall achievement at level 3 in a course in mathematics can be constructed by reading from top to bottom in the column of the achievement chart headed "70-79% (Level 3)".

Achievement Chart – Grades 9–10, Mathematics

Categories	50–59% (Level 1)	60–69% (Level 2)	70–79% (Level 3)	80–100% (Level 4)
Knowledge/ Understanding	The student:			
– understanding of concepts	– demonstrates limited understanding of concepts	– demonstrates some understanding of concepts	– demonstrates considerable understanding of concepts	– demonstrates thorough understanding of concepts
– performing algorithms	– performs only simple algorithms accurately by hand and using technology	– performs algorithms with inconsistent accuracy by hand, mentally, and using technology	– performs algorithms accurately by hand, mentally, and using technology	– selects the most efficient algorithm and performs it accurately by hand, mentally, and using technology
Thinking/Inquiry/ Problem Solving	The student:			
– reasoning	– follows simple mathematical arguments	– follows arguments of moderate complexity and makes simple arguments	– follows arguments of considerable complexity, judges the validity of arguments, and makes arguments of some complexity	– follows complex arguments, judges the validity of arguments, and makes complex arguments
– applying the steps of an inquiry/problem-solving process (e.g., formulating questions; selecting strategies, resources, technology, and tools; representing in mathematical form; interpreting information and forming conclusions; reflecting on reasonableness of results)	– applies the steps of an inquiry/problem-solving process with limited effectiveness	– applies the steps of an inquiry/problem-solving process with moderate effectiveness	– applies the steps of an inquiry/problem-solving process with considerable effectiveness	– applies the steps of an inquiry/problem-solving process with a high degree of effectiveness and poses extending questions
Communication	The student:			
– communicating reasoning orally, in writing, and graphically	– communicates with limited clarity and limited justification of reasoning	– communicates with some clarity and some justification of reasoning	– communicates with considerable clarity and considerable justification of reasoning	– communicates concisely with a high degree of clarity and full justification of reasoning
– use of mathematical language, symbols, visuals, and conventions	– infrequently uses mathematical language, symbols, visuals, and conventions correctly	– uses mathematical language, symbols, visuals, and conventions correctly some of the time	– uses mathematical language, symbols, visuals, and conventions correctly most of the time	– routinely uses mathematical language, symbols, visuals, and conventions correctly and efficiently
Application	The student:			
– applying concepts and procedures relating to familiar and unfamiliar settings	– applies concepts and procedures to solve simple problems relating to familiar settings	– applies concepts and procedures to solve problems of some complexity relating to familiar settings	– applies concepts and procedures to solve complex problems relating to familiar settings; recognizes major mathematical concepts and procedures relating to applications in unfamiliar settings	– applies concepts and procedures to solve complex problems relating to familiar and unfamiliar settings

Explanatory Notes

The following definitions of terms are intended to help teachers and parents/ guardians use this document. It should be noted that, where examples are provided, they are suggestions and are not meant to be exhaustive.

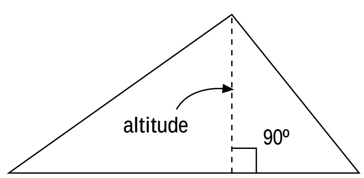
acute triangle. A triangle in which each of the three interior angles measures less than 90º.

algebraic expression. One or more variables and possibly numbers and operation symbols. For example, $3x + 6$, x, $5x$, and $21 – 2w$ are all algebraic expressions.

algebraic modelling. The process of representing a relationship by an equation or a formula, or representing a pattern of numbers by an algebraic expression.

algorithm. A specific set of instructions for carrying out a procedure.

altitude. A line segment giving the height of a geometric figure. In a triangle, an altitude is found by drawing the perpendicular from a vertex to the side opposite. For example:



analytic geometry. A geometry that uses the *xy*-plane to determine equations that represent lines and curves.

angle bisector. A line that divides an angle into two equal parts.

application. An area outside of mathematics within which concepts and skills of mathematics may be used to solve problems.

binomial. An algebraic expression containing two terms, for example, $3x + 2$.

centroid of a triangle. The point of intersection of the three medians of a triangle. Also called *balance point*.

chord. A line segment joining two points on a curve.

circumcentre of a triangle. The centre of the circle that passes through the three vertices of a triangle.

coefficient. The factor by which a variable is multiplied. For example, in the term $5x$, the coefficient is 5; in the term ax, the coefficient is a.

congruence. The property of being congruent. Two geometric figures are congruent if they are equal in all respects.

constant rate of change. A relationship between two variables illustrates a constant rate of change when equal intervals of the first variable are associated with equal intervals of the second variable. For example, if a car travels at 100 km/h, in the first hour it travels 100 km, in the second hour it travels 100 km, and so on.

curve of best fit. The curve that best describes the distribution of points in a scatter plot.

diagonal. In a polygon, a line joining two vertices that are not next to each other (i.e., not joined by one side).

difference of squares. A technique of factoring applied to an expression of the form $a^2 – b^2$, which involves the subtraction of two perfect squares.

direct variation. A relationship between two variables in which one variable is a constant multiple of the other.

dynamic geometry software. Computer software that allows the user to plot points on a coordinate system, measure line segments and angles, construct two-dimensional shapes, create two-dimensional representations of three-dimensional objects, and transform constructed figures by moving parts of them.

evaluate. To determine a value for.

exponent. A special use of a superscript in mathematics. For example, in 3^2, the exponent is 2. An exponent is used to denote repeated multiplication. For example, 5^4 means 5 x 5 x 5 x 5.

exponential notation. The notation used by calculators to display numbers that are too large or too small to fit onto the screen of the calculator. For example, the number 25 382 000 000 000 000 might appear as "2.5382 16" on a calculator screen. The digits "16" to the right of the expression indicate the number of places that the decimal point should be moved to express the number in normal form.

extrapolate. To estimate values lying outside the range of given data. For example, to extrapolate from a graph means to estimate coordinates of points beyond those that are plotted.

factor. To express a number as the product of two or more numbers, or an algebraic expression as the product of two or more other algebraic expressions. Also, the individual numbers or algebraic expressions in such a product.

finite differences. Given a table of values in which the x-coordinates are evenly spaced, the first differences are calculated by subtracting consecutive y-coordinates. The second differences are calculated by subtracting consecutive first differences, and so on. In a linear function, the first differences are constant; in a quadratic function, the second differences are constant. For example:

x	y	First Difference	Second Difference
1	1		
2	4	4 – 1 = 3	
3	9	9 – 4 = 5	5 – 3 = 2
4	16	16 – 9 = 7	7 – 5 = 2
5	25	25 – 16 = 9	9 – 7 = 2

first-degree equation. An equation in which the variable has the exponent 1. For example, $5(3x - 1) + 6 = -20 + 7x + 5$.

first-degree inequation. An inequality in which the variable has the exponent 1. For example, $6 + 2x + 8 \geq 4x + 20$.

first-degree polynomial. A polynomial in which the variable has the exponent 1. For example, $4x + 20$.

first differences. *See* **finite differences**.

function. A relation in which for each value of x there is only one value of y.

generalize. To determine a general rule or conclusion from examples. Specifically, to determine a general rule to represent a pattern or relationship between variables.

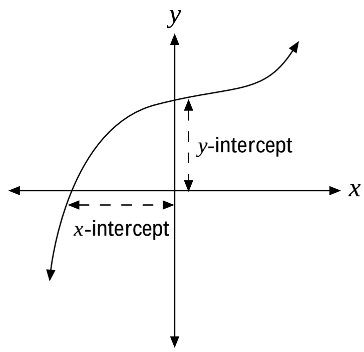
graphing calculator. A hand-held device capable of a wide range of mathematical operations, including graphing from an equation, constructing a scatter plot, determining the equation of a curve of best fit for a scatter plot, making statistical calculations, performing elementary symbolic manipulation. Many graphing calculators will attach to scientific probes that can be used to gather data involving physical measurements (e.g., position, temperature, force).

graphing software. Computer software that provides features similar to those of a graphing calculator.

infer from data. To make a conclusion based on a relationship identified between variables in a set of data.

integer. Any one of the numbers …, –4, –3, –2, –1, 0, +1, +2, +3, +4, …

intercept. The distance from the origin of the *xy*-plane to the point at which a line or curve cuts a given axis (e.g., *x*-intercept or *y*-intercept). For example:



interpolate. To estimate values lying between elements of given data. For example, to interpolate from a graph means to estimate coordinates of points between those that are plotted.

linear relation. A relation between two variables that appears as a straight line when graphed on a coordinate system. May also be referred to as a *linear function*.

linear system. A pair of equations of straight lines.

line of best fit. The straight line that best describes the distribution of points in a scatter plot.

make inferences from data. *See* **infer from data**.

manipulate. To apply operations, such as addition, multiplication, or factoring, on algebraic expressions.

mathematical model. A mathematical description (e.g., a diagram, a graph, a table of values, an equation, a formula, a physical model, a computer model) of a real situation.

mathematical modelling. The process of describing a real situation in a mathematical form. *See also* **mathematical model**.

measure of central tendency. A value that can represent a set of data, for example, the mean, the median, or the mode.

median. *Geometry.* The line drawn from a vertex of a triangle to the midpoint of the opposite side. *Statistics.* The middle number in a set, such that half the numbers in the set are less and half are greater when the numbers are arranged in order.

method of elimination. In solving systems of linear equations, a method in which the coefficients of one variable are matched through multiplication and then the equations are added or subtracted to eliminate that variable.

method of substitution. In solving systems of linear equations, a method in which one equation is rearranged and substituted into the other.

model. *See* **mathematical model**.

monomial. An algebraic expression with one term, for example, $5x^2$.

multiple trials. A technique used in experimentation in which the same experiment is done several times and the results are combined through a measure such as averaging. The use of multiple trials "smooths out" some of the random occurrences that can affect the outcome of an individual trial of an experiment.

non-linear relation. A relationship between two variables that does not fit a straight line when graphed.

non-real root of an equation. A solution to an equation that is not an element of the set of real numbers (e.g., $\sqrt{-16}$). *See* **real root of an equation**.

optimal value. The maximum or minimum value of a variable.

partial variation. A relationship between two variables in which one variable is a multiple of the other, plus some constant number. For example, the cost of a taxi fare has two components, a flat fee and a fee per kilometre driven. A formula representing the situation of a flat fee of $2.00 and a fee rate of $0.50/km would be $F = 0.50d + 2.00$, where F is the total fare and d is the number of kilometres driven.

piecewise linear function. A function composed of two or more linear functions having different slopes.

polygon. A closed figure formed by three or more line segments. Examples of polygons are triangles, quadrilaterals, pentagons, octagons.

polynomial. *See* **polynomial expression.**

polynomial expression. An algebraic expression of the form $a + bx + cx^2 + \ldots$, where a, b, and c are numbers.

population. *Statistics.* The total number of individuals or items under consideration in a surveying or sampling activity.

primary trigonometric ratios. The basic ratios of trigonometry (i.e., sine, cosine, and tangent).

prism. A three-dimensional figure with two parallel, congruent polygonal bases. A prism is named by the shape of its bases, for example, rectangular prism, triangular prism.

proportional reasoning. Reasoning or problem solving based on the examination of equal ratios.

Pythagorean theorem. The conclusion that, in a right triangle, the square of the length of the longest side is equal to the sum of the squares of the lengths of the two other sides.

quadratic equation. An equation that contains at least one term whose exponent is 2, and no term with an exponent greater than 2, for example, $x^2 + 7x + 10 = 0$.

quadratic formula. A formula for determining the roots of a quadratic equation, $ax^2 + bx + c = 0$. The formula is phrased in terms of the coefficients of the quadratic equation:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

quadratic function. A function whose equation is in quadratic form, for example, $y = x^2 + 7x + 10$.

quadrilateral. A polygon with four sides.

randomization. A principle of data analysis that involves selecting a sample in such a way that each member of the population has an equally likely chance of being selected.

rational number. A number that can be expressed as the quotient of two integers where the divisor is not 0.

realistic situation. A description of an event or events drawn from real life or from an experiment that provides experience with such an event.

real root of an equation. A solution to an equation that is an element of the set of real numbers. The set of real numbers includes all numbers commonly used in daily life: all fractions, all decimals, all negative and positive numbers.

region on the *xy*-plane. An area bounded by a curve or curves and/or lines on the *xy*-plane.

regression. A method for determining the equation of a curve (not necessarily a straight line) that fits the distribution of points on a scatter plot.

relation. An identified relationship between variables that may be expressed as a table of values, a graph, or an equation.

representivity. A principle of data analysis that involves selecting a sample that is typical of the characteristics of the population from which it is drawn.

right triangle. A triangle containing one 90° angle.

sample. A small group chosen from a population and examined in order to make predictions about the population.

sampling technique. A process for collecting a sample of data.

scatter plot. A graph that attempts to show a relationship between two variables by means of points plotted on a coordinate grid. Also called *scatter diagram*.

scientific probe. A device that may be attached to a graphing calculator or to a computer in order to gather data involving measurement (e.g., position, temperature, force).

second-degree polynomial. A polynomial in which at least one term, the variable, has an exponent 2, and for no term is the exponent of the variable greater than 2, for example, $4x^2 + 20$ or $x^2 + 7x + 10$.

second differences. *See* **finite differences**.

similar triangles. Triangles in which corresponding sides are proportional.

simulation. A probability experiment to estimate the likelihood of an event. For example, tossing a coin is a simulation of whether the next person you meet will be male or female.

slope. A measure of the steepness of a line, calculated as the ratio of the rise (vertical distance travelled) to the run (horizontal distance travelled).

spreadsheet. Computer software that allows the entry of formulas for repeated calculation.

substitution. The process of replacing a variable by a value. *See also* **method of substitution.**

system of equations. A system of linear equations comprises two or more equations in two or more variables. The solution to a system of linear equations in two variables is the point of intersection of two straight lines.

table of values. A table used to record the coordinates of points in a relation. For example:

x	$y = 3x - 1$
–1	–4
0	–1
1	2
2	5

variable. A symbol used to represent an unspecified number. For example, x and y are variables in the expression $x + 2y$.

vertex. A point at which two sides of a polygon meet.

vertical stretch factor. A coefficient in an equation of a relation that causes stretching of the corresponding graph in the vertical direction only. For example, the graph of $y = 3x^2$ would appear to be narrower than the graph of $y = x^2$ because its y-coordinates are three times as great for the same x-coordinate.

***xy*-plane.** A coordinate system based on the intersection of two straight lines called axes, which are usually perpendicular. The horizontal axis is the x-axis, and the vertical axis is the y-axis. The point of intersection of the axes is called the origin.

zeros of a function. The values of x for which a function has a value of zero. The zeros of a function correspond to the x-intercepts of its graph. *See also* **intercept**.

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